

Here's the solution to the derivation discussed in class.

1. At the root, use rule  $\supset E$ , to get two new nodes:  $- \models P(f(a)) \wedge f(a) = b$  and  $- \models [P(f(a)) \wedge f(a) = b] \supset P(b)$ .
2. The first of these is handled by  $\wedge I$  to produce two leafs.
3. For the second, use  $\forall E^*$  (the new rule) to get  $- \models \forall x[P(x) \wedge x = b] \supset P(b)$ .
4. Next reduce this to constants using  $\forall I$  to get  $- \models [P(c) \wedge c = b] \supset P(b)$ .
5. The rest is the usual (5 more nodes) without using function symbols: apply the rules  $\supset I$ , then  $= E$ , then twice  $\wedge E$ .