CSC 330F 2005, Assignment 5, St. George Campus
Due: Dec. 8th, 2:10 PM

On the cover page of your assignment, you must list everyone with whom you discussed this assignment, and which problems you discussed with each person. You must also write **and sign** *the following statement: "I have read and understood the policy on collaboration on homework assignments stated in the Course Information handout." Without these, your homework will not be marked.*

Imagine that we use a robot to push around boxes that are located in various rooms of a large factory. There are two types of boxes: small ones that are sealed, and large ones that may contain up to two small ones. There are doors between the factory rooms, and the robot can go from one room to another, provided that there is a door between them. It can also push a box and its contents through the door. While the robot is in a room, it can put a small box into a large one, and unload the contents of a large box.

We can describe this world in a first-order language with equality $\mathcal{L}$ that has three state-dependent predicates, $RobotLoc(r, s)$ (the robot is situated in room r), $BoxLoc(b, r, s)$ (box b is situated in room r), and $Contains(b_1, b_2, s)$ (large box b_1 contains small box b_2). The language $\mathcal{L}$ also has state-independent predicates: $Box(x)$, $Door(x)$, $Room(x)$ (object x is a box, door, or room respectively), $Connects(d, r_1, r_2)$ (door d connects room r_1 and room r_2), and $Small(b)$ (box b is small). We assume that if the formula $Connects(d, r_1, r_2)$ is true, then $Connects(d, r_2, r_1)$ is true as well.

For our purposes, we can assume that there are four transactions available to the robot: *PutIn* (put box b_1 into box b_2), *UnLoad* (remove all the contents of box b), *GoThru* (go through door d), and *PushThru* (push box b through door d).

1. Write sentences of $\mathcal{L}$ characterizing the following predicates: $PutInPrecond(b_1, b_2, s)$, $UnLoadPrecond(b, s)$, $GoThruPrecond(d, s)$, and $PushThruPrecond(b, d, s)$ (that is, the preconditions of the four transactions). [12]

2. Write sentences of $\mathcal{L}$ characterizing each of the following predicates: $PutInPostcond(b_1, b_2, s, s')$, $UnLoadPostcond(b, s, s')$, $GoThruPostcond(d, s, s')$, and $PushThruPostcond(b, d, s, s')$ (that is, the postconditions of the four transactions). [12]

3. Let F be the formula of $\mathcal{L}$ with one free variable s stating that in state s, each small box is in a large one. For which of the four transactions is F an invariant? Explain in each case what logical entailment you would need to prove or disprove to show this. (You do not need to prove anything.) [8]

4. Write a formula G of $\mathcal{L}$ with one free variable s that is invariant under the *GoThru* transaction but is not invariant under the *PushThru* transaction. Explain your answer briefly. [6]

5. Unloading a box does not change the location of the robot. Express this as a logical entailment and prove that it is a property of your formalization. [12]

Bonus question: Consider a variant of the robot world where instead of small and large boxes, there are boxes of various sizes, and a box may contain at most one other box provided that the former is larger than than the latter. We now use a language $\mathcal{L}'$ that is just like $\mathcal{L}$ but has a predicate $\mathit{Larger}(b_1, b_2)$ instead of $\mathit{Small}(b)$. Remarkably, it can be shown that there is no formula of $\mathcal{L}'$ that can express the postcondition of $\mathit{PushThru}$. Find out what the problem is.

Do not attempt any bonus work until the regular part of your assignment is complete.