PLEASE HAND IN

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UNIVERSITY OF TORONTO

FACULTY OF ARTS AND SCIENCE

MIDTERM JULY 2004

CSC 236H1 Y

DURATION — 50 MINUTES

NO AIDS ALLOWED

STUDENT NUMBER: |_|_|_|_|_|_|_|_|_|

LAST NAME: ______________________________

FIRST NAME: ______________________________

Do NOT *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This exam consists of 3 questions on 4 pages (including this one) *When you receive the signal to start, please make sure that your copy of the examination is complete.*

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on. You will earn substantial part marks for writing down the outline of a proof and indicating which steps are missing.

Write your student number at the bottom of pages 2-4 of this test.

1: ______/ 5

2: ______/ 5

3: ______/ 5

TOTAL: ______/15

Good Luck!

QUESTION 1. [5 MARKS]

Define $g(n)$ as:

$$g(n) = \sum_{i=0}^{2n} 5^i.$$

Prove that $g(n) \bmod 6$ is equal to 1 for all $n \in \mathbb{N}$. In other words, prove that for each $n \in \mathbb{N}$, there is some integer k such that $g(n) = 6k + 1$.

Question 2. [5 marks]

For $k \in \mathbb{N}$, define $U(3^k)$ as:

$$U(3^k) = \begin{cases} c, & k = 0 \\ 3U(3^{k-1}) + d3^k, & k > 0 \end{cases}.$$

Prove that for all $k \in \mathbb{N}$, $U(3^k) = 3^k(c + dk)$.

QUESTION 3. [5 MARKS]

Let $PV = \{v, w, x, y, z\}$ be a set of propositional variables. Define a special set of propositional formulas $\mathcal{F}^*$ as the smallest set such that

BASIS: Any propositional variable in PV belongs to $\mathcal{F}^*$.

INDUCTION STEP: If P_1 and P_2 belong to $\mathcal{F}^*$, then so do $(P_1 \wedge P_2)$, $(P_1 \vee P_2)$, $(P_1 \rightarrow P_2)$ and $(P_1 \leftrightarrow P_2)$.

For a propositional formula f, define $\mathbf{cn}(f)$ as the number of instances of connectives from $\{\vee, \wedge, \rightarrow, \leftrightarrow\}$ in f. Define $\mathbf{p}(f)$ as the number of parentheses in f.

Use structural induction to prove that for all $f \in \mathcal{F}^*$, $\mathbf{p}(f) = 2\mathbf{cn}(f)$.

Total Marks = 15