CSC236 quiz 5, Tuesday June 21st

Name: Student number:

Suppose that f and l are integers, that $l > f+1$, and that $m = \lfloor (f+l)/2 \rfloor$ ($\lfloor x \rfloor$ is the greatest integer no bigger than x, also called the floor of x). Prove that $f < m < l$ (no induction required).

Proof: By assumption l and $f+1$ are integers, so $l > f+1$ implies that l is at least $f+2$. This means that

$$\begin{aligned} m &= \left\lfloor \frac{f+l}{2} \right\rfloor && \text{[by definition]} \\ &\leq \left\lfloor \frac{l-2+l}{2} \right\rfloor && \text{[since } f \leq l-2\text{]} \\ &= \left\lfloor \frac{2l-2}{2} \right\rfloor = \lfloor l-1 \rfloor = l-1 && \text{[defn. of floor, since } l-1 \text{ is an integer]} \\ &< l \end{aligned}$$

Thus $m < l$. On the other hand

$$\begin{aligned} m &= \left\lfloor \frac{f+l}{2} \right\rfloor && \text{[by definition]} \\ &\geq \left\lfloor \frac{f+f+2}{2} \right\rfloor && \text{[since } l \geq f+2\text{]} \\ &= \left\lfloor \frac{2f+2}{2} \right\rfloor = \lfloor f+1 \rfloor = f+1 && \text{[defn. of floor, since } f+1 \text{ is an integer]} \\ &> f. \end{aligned}$$

Thus $f < m$. Combining the two results, $f < m < l$. QED.