CSC236 QUIZ 3, TUESDAY JUNE 7TH

Name: Student number:

Using simple induction, prove that

$$\left(\sum_{t=0}^{n} 2 \cdot 3^t\right) = 3^{n+1} - 1$$

... is true for all $n \in \mathbb{N}$.

SAMPLE SOLUTION: Let $P(n)$ be "$\sum_{t=0}^{n} 2 \cdot 3^t = 3^{n+1} - 1$." Then for all $n \in \mathbb{N}$, $P(n)$.

PROOF (INDUCTION ON n): Suppose $n = 0$. Then $P(n)$ claims that $2 \cdot 3^0 = 2 = 3^1 - 1$, which is certainly true, so the base case, $P(0)$ is verified.

INDUCTIVE STEP: For some arbitrary $n \in \mathbb{N}$, assume that $P(n)$ is true. I must show that this implies $P(n+1)$. The sum $\sum_{t=0}^{n+1} 2 \cdot 3^t$ can be broken up as

$$\begin{aligned} \left(\sum_{t=0}^{n} 2 \cdot 3^t\right) + 2 \cdot 3^{n+1} &= 3^{n+1} - 1 + 2 \cdot 3^{n+1} \qquad \text{(by IH)} \\ &= 3 \cdot 3^{n+1} - 1 = 3^{n+2} - 1. \end{aligned}$$

This is exactly what $P(n+1)$ claims, so $P(n)$ implies $P(n+1)$.

I conclude that $P(n)$ is true for all $n \in \mathbb{N}$. QED.