Using simple induction, prove that
\[
\left( \sum_{i=0}^{n} 2 \cdot 3^i \right) = 3^{n+1} - 1
\]
... is true for all \( n \in \mathbb{N} \).

**Sample solution:** Let \( P(n) \) be "\( \sum_{i=0}^{n} 2 \cdot 3^i = 3^{n+1} - 1 \)." Then for all \( n \in \mathbb{N} \), \( P(n) \).

**Proof (induction on \( n \)):** Suppose \( n = 0 \). Then \( P(n) \) claims that \( 2 \cdot 3^0 = 2 = 3^1 - 1 \), which is certainly true, so the base case, \( P(0) \) is verified.

**Inductive step:** For some arbitrary \( n \in \mathbb{N} \), assume that \( P(n) \) is true. I must show that this implies \( P(n + 1) \). The sum \( \sum_{i=0}^{n+1} 2 \cdot 3^i \) can be broken up as
\[
\left( \sum_{i=0}^{n} 2 \cdot 3^i \right) + 2 \cdot 3^{n+1} = 3^{n+1} - 1 + 2 \cdot 3^{n+1} \quad \text{(by IH)}
= 3 \cdot 3^{n+1} - 1 = 3^{n+2} - 1.
\]

This is exactly what \( P(n + 1) \) claims, so \( P(n) \) implies \( P(n + 1) \).

I conclude that \( P(n) \) is true for all \( n \in \mathbb{N} \). QED.