CSC236 quiz 2, Tuesday May 31st

Name: Student number:

Using simple induction on n, prove that if $m, n \in \mathbb{N}$, then $m^n - 1$ is an integer multiple of $(m-1)$. Notice that induction on m is not necessary, and define $0^0 = 1$ (If this definition troubles you, just use it for this quiz and ask me about it later).

Sample solution: Claim: Let $P(n)$ be "If m is a natural number, then $m^n - 1$ is an integer multiple of $(m-1)$." Then $\forall n \in \mathbb{N}, P(n)$.

Proof (induction on n): $P(0)$ says that $m^0 - 1$ is an integer multiple of $(m-1)$, which is certainly true since for all natural numbers m, $m^0 - 1 = 0$, which is a multiple of any integer, including $m - 1$. Thus the base case holds.

Induction step: Assume that $P(n)$ holds for some arbitrary natural number n. So there exists an integer k such that $m^n = (m-1)k + 1$. We can write m^{n+1} as $m \times m^n$, which (by the IH) equals $m((m-1)k+1) = (m-1)(mk+1)+1$. Since the integers are closed under multiplication and addition, $mk + 1$ is an integer whenever m, k and 1 are, so $m^{n+1} - 1 = (m-1)(mk+1)$, an integer multiple of $m - 1$. Hence $P(n+1)$ is true, and we have shown that $P(n) \Rightarrow P(n+1)$ for an arbitrary natural number n.

Conclude, by induction, that $P(n)$ is true for all natural numbers n. QED.

Marking scheme: 1 mark for verifying the base case, 2 marks for the induction step, which should assume the proposition for an arbitrary n and show that this assumption implies the proposition for $n + 1$. One mark for concluding $P(n) \forall n \in \mathbb{N}$.