CSC236 QUIZ 10, TUESDAY AUGUST 8

Name: Student number:

Let $\Sigma = \{0, 1\}$, and $L = \{x \in \Sigma^* : x \text{ has at most two 0s}\}$. Prove that $L = L(1^*(0+\varepsilon)1^*(0+\varepsilon)1*)$. HINT: You must prove that each language includes the other as a subset.

CLAIM: $L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^* \subseteq L$.

PROOF: Let x be an arbitrary element of $L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$. By definition $L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*) = L(1^*)L(0 + \varepsilon)L(1^*)L(0 + \varepsilon)L(1^*)$, so $x = uvwyz$, where $u, w, z \in L(1^*)$, and $v, y \in L(0 + \varepsilon)$. Thus u, w, z contain no 0s, and v, y contain, at most, one 0 each, for a total of (at most) two 0s. So $x \in L$, and since x was chosen arbitrarily, $L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*) \subseteq L$. QED.

CLAIM: $L \subseteq L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$.

PROOF: Let x be an arbitrary element of L. If x has no zeros, then $x = \varepsilon\varepsilon\varepsilon\varepsilon t$, where $t \in L(1^*)$, and (since $\varepsilon \in L(1^*)$ and $\varepsilon \in L(0 + \varepsilon)$) $x \in L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$. If x has one zero, then $x = u0\varepsilon\varepsilon t$, where $u, t \in L(1^*)$, and (since $\varepsilon \in L(1^*)$ and $\varepsilon, 0 \in L(0+\varepsilon)$), $x \in L(1^*(0+\varepsilon)1^*(0+\varepsilon)1^*)$. If x has two zeros, then $x = u0v0y$, where $u, v, y \in L(1^*)$ and (since $0 \in L(0 + \varepsilon)$), $x \in L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$. In all cases $x \in L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$, and (since x was chosen arbitrarily) $L \subseteq L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$. QED.

CLAIM: $L = L(1^*(0 + \varepsilon)1^*(0 + \varepsilon)1^*)$.

PROOF: In the previous claims I showed mutual inclusion, so the two languages are equal (as sets).