CSC236 quiz 9, Tuesday July 26

Name: Student number:

Suppose $\mathcal{L}$ is a first-order language that contains the following formula:

$$\forall x \forall y \forall z (\neg L(x,z) \Rightarrow (\neg L(y,z) \vee \neg L(x,y)).$$

Give one interpretation, $\mathcal{I}_1$, that satisfies the formula, and one interpretation, $\mathcal{I}_2$, that falsifies the formula.

Sample solution: Let $\mathcal{I}_1 = (S_1, \sigma_1)$, where S_1 has the domain D the set of integers, and $L(x,y)$ means $x < y$, no constants are defined, and σ_1 is not specified. Then the formula is simply the contrapositive of:

$$\forall x \forall y \forall z ((L(x,y) \wedge L(y,z)) \rightarrow L(x,z)),$$

which is simply the transitive property of the $<$ relation.

Let $\mathcal{I}_2 = (S_2, \sigma_2)$, where S_2 has the domain D of all people, and $L(x,y)$ means that x likes y. There are examples of 3 people where x likes y, y likes z but x doesn't like (or perhaps even know) z.