CSC236 QUIZ 8, TUESDAY JULY 19

Name: Student number:

1. Using only logical equivalences (Law of double negation, De Morgan's laws, commutative law, associative law, distributive law, identity law, idempotency law, $\rightarrow$ law, $\leftrightarrow$ law), but no truth table, prove the following:

$$(x \rightarrow y) \land (x \rightarrow z) \land (x \rightarrow w) \text{ LEQV } x \rightarrow (y \land z \land w).$$

SAMPLE SOLUTION:

$$\begin{aligned}(x \rightarrow y) \land (x \rightarrow z) \land (x \rightarrow w) \quad &\text{LEQV} \quad (\neg x \lor y) \land (\neg x \lor z) \land (\neg x \lor w) \quad [\rightarrow \text{law}]\\ &\text{LEQV} \quad (\neg x \lor y) \land (\neg x \lor (z \land w)) \quad [\text{Distributive and associative laws}]\\ &\text{LEQV} \quad \neg x \lor (y \land z \land w) \quad [\text{Distributive law}]\\ &\text{LEQV} \quad x \rightarrow (y \land z \land w) \quad [\rightarrow \text{law}]\end{aligned}$$

2. Write a DNF (DISJUNCTIVE NORMAL FORM) formula equivalent to $x \rightarrow (y \land z)$.

SAMPLE SOLUTION: Apply the $\rightarrow$ rule, and $\neg x \lor (y \land z)$ is a DNF formula, since is is a disjunction of minterms (minterms are conjunctions of 1 or more literals).

A slightly longer, although equally good, solution is to examine the truth table, and create conjunctions of literals that are equivalent to the truth assignments that satisfy the formula:

x	y z	$x \rightarrow (y \land z)$		
0	0	0	1	$(\neg x \land \neg y \land \neg z)$
0	0	1	1	$(\neg x \land \neg y \land z)$
0	1	0	1	$(\neg x \land y \land \neg z)$
0	1	1	1	$(\neg x \land y \land z)$
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	$(x \land y \land z)$

Tying these together gives the DNF formula:

$$(\neg x \land \neg y \land \neg z) \lor (\neg x \land \neg y \land z) \lor (\neg x \land y \land \neg z) \lor (\neg x \land y \land z) \lor (x \land y \land z)$$