Assignment 3
marking scheme

1. This question was marked out of 5, and then scaled by 30 (so multiply your mark by 6). Three marks for having the structure of correctness proof, two marks for getting the details right. Automatic one-mark deduction for claiming that the difference of two natural numbers is a natural number (you can get this mark back if you have a convincing argument that $5 - 6$ is a natural number).

2. This question was marked out of 16, then scaled to 20 (so multiply your mark by 20/16).

   - -1 if predicate was incorrect e.g. if didn't mention termination, or not with respect to p, or not for every b.
   - -1 if didn't use complete induction
   - -1 if induction hypothesis malformed
   - -1 mark for omitting any of the three cases (1 mark off for each missed case)
   - $p = 0$ case: -1 for not relating the argument back to the program's code
   - $p = 1$ case: -1 for not relating the argument back to the program's code, -1 for failing to handle termination
   - $p > 1$ case: -1 for missing either of the two cases (1 mark off for each case)

     ODD CASE: -1 for not relating argument back to the program's code, -1 for failing to handle termination, -1 for failing to note that $(p-1)/2 = p/2$ in this case

     EVEN CASE: -1 for not relating argument back to the program's code, -1 for failing to handle termination.

   This question was well done. A few failed to note that $(p-1)/2 = p/2$ in the odd case. A handle just made claims along the lines of, "the program returns x in this case" without relating things back to the code to justify why x was returned. I wasn't looking for a lot of justification, just a line or two saying p is odd or $p < 2$ etc. One or two people confused recursive and iterative programs and attempted to come up with a loop invariant and prove termination separately.

3. This question was marked out of 5, and then scaled to 30, so multiply your mark by 6. Three marks for having the correct structure of the proof, two marks for the details.

4. This question was marked out of 30, and scaled to 20 (so multiply your mark by 2/3).

   GENERAL: -1 if any of the four parts was missing, -1 if any extraneous parts were added

   PROVING THE CLOSED FORM: -1 if formula incorrect, -1 for mistake in base case -1 for malformed induction hypothesis -1 for failing to use the definition of T to expand $T(5^k + 1)$ in induction step, -1 for failing to use the I.H. in I.S., -1 for failing to conclude.

PROVING FLOOR/CEILING INEQUALITY: -1 for failing to show $\lfloor n/5 \rfloor \leq \lceil n/5 \rceil$, -1 for failing to show $\lfloor n/5 \rfloor \geq 1$, -1 for not breaking $\lceil n/5 \rceil$ into two cases, -1 for not showing $\lceil n/5 \rceil < n$ in each of the two cases.

PROVING MONOTONICITY: -1 for incorrect predicate, -1 if any of the three cases ommitted (1 mark each), $n = 5$ case: -1 for failing to use definition of T properly. $n > 5$ case: -1 for not using previous lemma to observe $1 \leq \lfloor n/5 \rfloor, \lceil n/5 \rceil, n-1 < n$ and therefore failing to justify why allowed to use I.H. -1 for not noticing (and justifying) reduces to $n-1 < n-1$ for not using $T(n-1)$ definition for $n-1 \geq 5$. -1 for not using I.H. for $T(\lfloor n/5 \rfloor$ and $T(\lceil n/5 \rceil)$. -1 for failing to conclude.

FINDING THE BOUND: -1 for missing $n = 5^{\log_5 n}$. -1 for not having $5^{\log_5 n} < 5^{\lceil \log_5 n \rceil}$. -1 for not using monotonicity result, -1 for not using closed form result, -1 for mistake in algebra, -1 for not specifying κ.

Many people did not prove the floor and ceiling inequality and failed to justify why they could use their I.H. in the proof of monotonicity and lost either 5 or 6 marks as a result. This was the most common mistake. Otherwise, this question was reasonably well done.