Marking scheme
CSC236, Summer 2005, Assignment 1

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The general marking scheme structure is that you receive about 60% of the marks for structure, for showing you know what belongs where in a valid proof, and about 40% of the marks for detail, for clear exposition of the parts needed for a valid proof.

1. Parts (a) and (b) were each worth 5 marks, for a total of 10.

2. Part (a) was worth 5 marks, part (b) was worth 8, for a total of 13. The 8 marks in part (b) were grouped into 5 marks for proving that every group of 6 students contained at least 3 mutual strangers or 3 mutual acquaintances, and 3 marks for a counterexample showing that this property doesn’t hold for 5 students.

3. 6 marks in total. 5 marks for proving that your expression correctly counts the number of tilings of a $2 \times n$ space, 1 mark for providing a closed form.

4. Parts (a) and (b) were each marked out of 5. In part (a) there were 2 marks for defining a set (1 for knowing to define a set, 1 for the correctness of the set), 1 mark for checking that the set is non-empty, and 2 marks for showing that $r < n$ if they choose the smallest element of the set.

A lot of people assumed that $m = qn + r$ for some $q$ and $r$, then defined a set in terms of $q$ and $r$, showed it was non-empty (or attempted to show this), applied the well-ordering principle to get a smallest element, and then miraculously concluded that this smallest element was the $r$ they had assumed to exist above. Their set usually assumed that $r < n$, so they had nothing left to show at this point. They didn’t seem to realize they had assumed precisely what was to be proven.

Others tried to use proposition 1.7 to conclude the existence of suitable $q$ and $r$.

Many specified their sets imprecisely, using words with ambiguous intention. It was left to me to guess what they meant, though I always gave them the benefit of the doubt.

Most knew enough to define some set and attempted to show it was non-empty, and earned 2/5 for doing so.

Part (b) was marked out of 5 so that anyone leaving it blank got 1, and most of those who attempted it got full marks.

5. This question was marked out of 10: 2 marks for the base case (1 for knowing enough that it was needed, 1 for using $n = 0$), 2 for proper statement of induction hypothesis, 2 for starting with an arbitrary graph and removing a vertex (1 for starting with something large and making it smaller), 3 for the induction argument, 1 for the concluding statement.

Three or four knew exactly what to do and nearly produced the sample solution, although they may have lost a mark for a minor omission. Some people knew to start with $n + 1$ and remove a vertex, but did not consider that the graph could become disconnected. These people usually ended up with 9. Everyone else made the mistake of starting with a graph on $n$ vertices and trying to build a graph
on \( n + 1 \) vertices from this. These people were able to get 8/10 if this was the only mistake in their argument, but most also forgot the base case and ended up with 7.

6. This question was marked out of 5 (and this mark was doubled by Danny Heap to bring the total marks for question 6 to 10). The 5 marks were distributed as 1 mark for the basis, 1 for the induction hypothesis, 2 for the induction argument, and 1 mark for the concluding statement. In general this question was well done.