# CSC236, Summer 2005, Assignment 1

Due: Thursday June 9th, 10 am

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## Instructions

Please work on all questions. Turn in the outline and structure of a proof, even if you cannot provide every step of the proof, and we will try to assign some part marks. However, if there is any question you cannot see how to even begin, leave it blank you will receive 20% of the marks for that question.

Be sure to give full credit to any sources you consult (other than course notes, TAs, and the instructor) in preparing this problem set. If you try to pass off somebody else's work as your own for credit, you are committing an academic offense, and that can entail serious consequences. Any ideas that you do not attribute to someone else are assumed to be the ideas of the author(s) listed below, and will be evaluated for grading.

Write your name(s) and student number(s) (maximum of two names and two student numbers) in the space below.

Name ______________________________

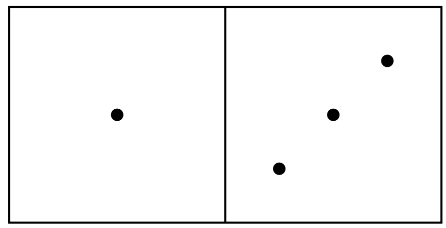
Student # ______________________________

Name ______________________________

Student # ______________________________

1. Count dominoes and drominoes:

   (a) A set of dominoes has 28 tiles, one for each unordered pair of dot sets, where the dot sets run from size zero (or blank) to six dots. How many tiles would there be in an extended game of dominoes, where the sizes of the dot set sizes run from 0 to $n$? Prove your claim. (If you've never played dominoes, consult your instructor or TA). Here's a sketch of the domino for the unordered pair $\{1, 3\}$:

   (b) Drominoes are like dominoes, except each tile is marked with an unordered triple instead of an unordered pair. How many dromino tiles are there if the triples run from $\{0, 0, 0\}$ to $\{6, 6, 6\}$? How about $\{0, 0, 0\}$ to $\{n, n, n\}$ (where $n$ is an arbitrary natural number)? Prove your claims.

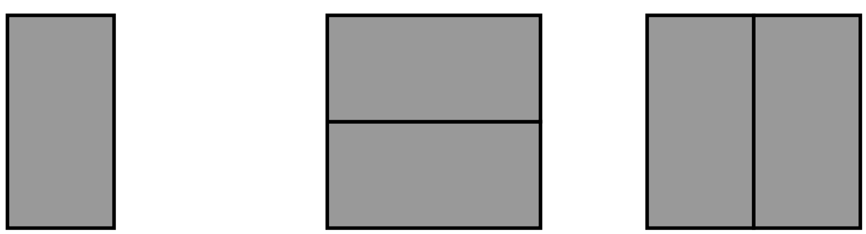
2. Count acquaintances:

   (a) On the last day of lecture of last Summer's CSC236 evening course, there were 51 students. Prove that there were at least two students with exactly the same number of student acquaintances attending the lecture.

   (b) During the break on the last day of lecture of last Summer's CSC236, more than six students lined up for beverages. Prove that among the first six students in the lineup there were either three mutual strangers or three mutual acquaintances (you are not allowed to assume that students are guaranteed to become acquaintances simply by standing in the same lineup). Is it also true that among the first five students in the line up there were either three mutual strangers or three mutual acquaintances? Prove your claim.

3. Tiling with dominoes

   Wooden dominoes are $2 \times 1$ rectangles, and if you turn them face down they have no identifying dots. In the diagram below I show (left) the one way to tile a $2 \times 1$ space with face-down dominoes, and (right) the two ways to tile a $2 \times 2$ space with dominoes.

   Call the number of ways to tile a $2 \times n$ space with dominoes $D(n)$. Find a (preferably closed form) expression for $D(n)$. A closed form involves a fixed (not depending on $n$) number of well-understood operations. Prove that your form is correct.

4. DIVISION ALGORITHM:

   (a) As part of Proposition 1.7 of the Course Notes there is a proof by induction that if $m, n \in \mathbb{N}$, and $n \neq 0$ then there are natural numbers $q$ and $r$ such that $m = qn + r$ and $r < n$. Use the Well-Ordering Principle for a different proof of this fact. HINT: Compare the equation $m = 0n + m$ to the equation $m = qn + r$. Notice that you aren't asked to prove the uniqueness of $q$ and $r$.

   (b) Read the proof of Proposition 1.7, and notice that it is CONSTRUCTIVE — that is if you provide a natural number $m$ and a positive natural number $n$, it tells you how to find a quotient $q$ and a remainder $r$ that satisfy $m = qn + r$, $0 \leq r < n$. Write a recursive java program that takes $m$ and $n$ as parameters and returns the pair $(q, r)$. The structure of your program should mimic the proof of Proposition 1.7.

5. CONNECTIONS

   In Section 0.5 you will find a definition of an undirected graph, in terms of its points and edges, as well as a definition of a PATH from point $x$ to point $y$. A CONNECTED GRAPH is an undirected graph which has a path from $x$ to $y$ for every pair of points $(x, y)$. Use complete induction to prove that a connected graph with $n$ points has at least $n - 1$ edges.

6. BIG NUMBERS

   For natural number $n$ we define:

$$n! = \begin{cases} 1, & n = 0 \\ 1, & n = 1 \\ n(n-1)!, & n > 1 \end{cases}.$$

   Use simple induction to prove that for all positive natural numbers $n$,

$$n! > \left(\frac{n}{e}\right)^n.$$

   You may assume, without proof, that for any positive natural number $n$,

$$\left(1 + \frac{1}{n}\right)^n < e.$$