CSC236, week 2 tutorial hints

May 25, 2004

Here are some further hints meant to help with assignment 1.

Questions 1–3: Read Example 1.1 (pages 22 and 23 of the Course Notes), which proves that the sum of the first \( n \) integers is \( n(n+1)/2 \). Try to to find a closed expression for the sum of the first \( n \) squares and the first \( n \) cubes.

Question 5: Read Theorem 5.13 (page 128 of the Course Notes). Modify this proof so that it uses well-ordering, in the following way. Without assuming that \( m \) and \( n \) are in least terms, show that if \( m \) and \( n \) are natural numbers such that \( m/n = \sqrt{2} \), then \( m/2 \) and \( n/2 \) are also natural numbers, as are \( m/4 \) and \( n/4 \). Iterate this reasoning to produce a non-empty set of natural numbers with no least element (a contradiction).

Questions 6–7: Turn this sketch into a proof that for each odd natural number \( n \), \( 4^n = 4 \pmod{5} \)

Base case: Verify that the claim works for the base case, when \( n = 1 \).

Induction step: If the claim holds for the \( n \), an odd number, show that the claim holds for \( n + 2 \).

Part of the challenge is writing up the proof is dealing with notation for odd numbers, so that each induction step increments by 2 rather than 1.