CSC165 Quiz 7, Thursday July 14th

Name: Student number:

Suppose $j, k \in \mathbb{N}$, $25 \leq j \leq 200$, $3200 \leq k \leq 102400$. Suppose j has binary representation $(b_n \cdots b_0)_2$, and k has binary representation $(b'_m \cdots b'_0)_2$. For the following questions, reading symbols from a calculator doesn't constitute justification.

1. What are the possible values of n and m? Justify your answer.

Sample solution: Since $25 \leq j \leq 200$, j's binary representation has at least as many digits as the binary representation of 25 (or $(11001)_2$, since $25 = 16 + 8 + 1$), which is 5 digits, and at most the number of binary digits as 25×8, which shifts the binary representation of 25 3 times to the left, so 8 digits. Thus n can have values from 4 to 7, inclusive (1 less than the number of digits). Since $32000 \leq k \leq 102400$, it has a mininum of the number of digits in 200×16, which is 200 shifted left 4 times, or 12 digits. It has a maximum of the number of digits in 102400, or 3200×32, which is 3200 shifted left 5 times, or 17 digits. Thus $11 \leq m \leq 16$ (the number of digits minus 1).

2. How many bits could there be in $9j/4$? Justify your answer.

Sample solution: $9j$ is calculated by adding $8j$ to j, which means shifting j to the left 3 times and then adding the result to j. The sum $j + 8j$ is no more than $2 \times 8j$, so the result will be at most 1 digit more than $8j$. Dividing by 4 shifts to the right 2 places, losing 2 digits. We calculated above that j has from 5 to 8 digits, so $9j$ has from 8 to 12 digits, and $9j/4$ has from 6 to 10 digits.