CSC165 Quiz 6, Thursday July 7th

Name: Student number:

1. Denote the real numbers by $\mathbb{R}$ and the integers by $\mathbb{Z}$. For real number x, define $\lfloor x \rfloor$ as the largest integer that is no greater than x. Recall from lecture that an equivalent definition of $\lfloor x \rfloor$ is: $y = \lfloor x \rfloor$ if and only if $y \in \mathbb{Z} \wedge y \leq x \wedge (\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y)$. Prove that $\forall x \in \mathbb{R}, \lfloor x \rfloor > x - 1$

   Sample solution:

   Let $x \in \mathbb{R}$

   Let $y = \lfloor x \rfloor$

   Let $z = y + 1$

   Then $z \in \mathbb{Z}$ ($y \in \mathbb{Z}$ and integers closed under addition).

   Also, $z \not\leq y$ (by construction of z)

   So $z \not\leq x$ (from the contrapositive of $z \leq x \Rightarrow z \leq y$ in the definition of $\lfloor x \rfloor$)

   So $x < z = y + 1 = \lfloor x \rfloor + 1$ ($z \not\leq x$ means $z > x$)

   So $x - 1 < \lfloor x \rfloor$ (subtracting 1 from both sides of the ineqality).

   Since x is an arbitrary element of $\mathbb{R}$, $\forall x \in \mathbb{R}$, $\lfloor x \rfloor > x - 1$.