

# CSC165 QUIZ 5, THURSDAY JUNE 23RD

Name:

Student number:

Suppose  $P$ ,  $Q$ ,  $R$ , and  $S$  are statements

1. Show that:

$$P \Rightarrow (Q \Rightarrow (R \Rightarrow S)),$$

... is equivalent to

$$(P \wedge Q \wedge R) \Rightarrow S.$$

SAMPLE SOLUTION:

$$\begin{aligned} P \Rightarrow (Q \Rightarrow (R \Rightarrow S)) &\Leftrightarrow \neg P \vee (\neg Q \vee (\neg R \vee S)) && \text{[transform } \Rightarrow \text{ to } \neg \text{ and } \vee] \\ &\Leftrightarrow (\neg P \vee \neg Q \vee \neg R) \vee S && \text{[associativity of } \vee] \\ &\Leftrightarrow \neg(P \wedge Q \wedge R) \vee S && \text{[DeMorgan's Law]} \\ &\Leftrightarrow (P \wedge Q \wedge R) \Rightarrow S && \text{[transform } \neg \text{ and } \vee \text{ to } \Rightarrow]. \end{aligned}$$

2. Show that:

$$((P \Rightarrow Q) \Rightarrow R) \Rightarrow S,$$

... is equivalent to

$$(\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S.$$

SAMPLE SOLUTION:

$$\begin{aligned} ((P \Rightarrow Q) \Rightarrow R) \Rightarrow S &\Leftrightarrow \neg(\neg(\neg P \vee Q) \vee R) \vee S && \text{[transform } \Rightarrow \text{ to } \neg \text{ and } \vee] \\ &\Leftrightarrow ((\neg P \vee Q) \wedge \neg R) \vee S && \text{[DeMorgan's Law]} \\ &\Leftrightarrow (\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S && \text{[distributivity of } \wedge] \end{aligned}$$