# CSC165 Quiz 10, Thursday August 11th

Name: Student number:

Suppose you have a floating-point representation that has $\beta = 10$ (base, or radix, 10), $t = 5$ digits, and $e \in \{-5, 5\}$. Thus a number is represented as $d_0 \cdot d_1 d_2 d_3 d_4 \times 10^e$, where $d_0 \neq 0$, $d_i \in \{0, \ldots, 9\}$, and $e \in \{-5, \ldots, 5\}$. . Suppose $x$ and $y$ are non-zero numbers that are represented by $x'$ and $y'$ (respectively) without overflow in your representation, using round-to-nearest. In the questions below, you may express your result as a fraction, or use a calculator.

1. What is the largest possible relative error, $|x - x'|/|x|$. Explain your answer.

   In Annie Yuk's tutorial, the word "largest" was spelled "smallest," so the answer was 0 (if $x$ could be perfectly represented as $x'$).

   Otherwise, The smallest numerator, for a given exponent $e$, is $x \geq 1.0000 \times 10^e$. Since $x$ is rounded to the nearest value, it differs from $x'$ by, at most, half the increment of the last digit, or $0.0001 \times 10^e/2$ $= 10^{e-4}/2$. This means that the relative error is, at most, $10^{-4}/2$, or .005%.

2. If $x = 3.0000500000$ and $y = 3.0000499999$, what is the relative error $|(x - y) - (x' - y')|/|x - y|$? Show your work.

   $x'$ rounds up $x$, so $x' = 3.0001$, $y'$ rounds down $y$, so $y' = 3.0000$, so $x' - y' = 0.0001$, whereas $x - y$ $= 0.0000000001$, so the relative error is

$$\frac{|0.0000000001 - 0.0001|}{|0.0000000001|} = 999999 = 99999900\%$$

3. What is the relative error if, in the previous part, we increase precision from $t = 5$ to $t = 8$ digits? Show your work.

   In this case $x'$ represents $x$ perfectly, so $x' = 3.0000500$, while $y'$ rounds $y$ up, so $y' = 3.0000500$. Thus, $x' - y' = 0.0000000$, and the relative error is

$$\frac{|0.0000000001|}{|0.0000000001|} = 1 = 100\%$$