CSC165, Summer 2005, Assignment 4

Due: Thursday July 28th, 10 am

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Instructions

Please work on all questions. Turn in the outline and structure of a solution, even if you cannot provide every step, and we will try to assign some part marks. However, if there is any question you cannot see how to even begin, leave it blank and you will receive 20% of the marks for that question.

Be sure to give full credit to any sources you consult (other than course notes, TAs, and the instructor) in preparing this problem set. If you try to pass off somebody else's work as your own for credit, you are committing an academic offense, and that can entail serious consequences. Any ideas that you do not attribute to someone else are assumed to be the ideas of the author(s) listed below, and will be evaluated for grading.

Write your name(s) and student number(s) (maximum of two names and two student numbers) in the space below.

Name ______________________________

Student # ______________________________

Name ______________________________

Student # ______________________________

1. Complete the methods `toBiNeg(int n)`, `toInt(String s)`, and `compare(String bn1, String bn2)` from `BiNegUtil.java` (on the web page). Do not import any packages, and do not convert `bn1` or `bn2` to integers in `compare`.

2. Consider the base -2 representation, where $b_i \in \{0, 1\}$, and

$$(b_n b_{n-1} \cdots b_1 b_0)_{-2} = \sum_{i=0}^{n} b_i (-2)^i.$$

   (a) How many positive integers can be written in an n-digit base -2 representation (including representations with leading zeros on the left)? How many negative numbers can be written in an n-digit base -2 representation (including representations with leading zeros on the left). Justify your answer.

   (b) For natural number k, what is the minimum number of digits needed to represent k in base -2 representation? Justify your answer.

3. Examine the method `mult(m,n)` in `BiNegUtil.java` (on the web page). Prove or disprove that if the loop invariant is true at the beginning of a loop iteration, then it is true at the end of a loop iteration. Your proof should be in the structured proof format from class.

4. Let $f : \mathbb{N} \to \mathbb{R}^{\geq 0}$, and let

$$O(f) = \{g : \mathbb{N} \to \mathbb{R}^{\geq 0} | \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \to g(n) \leq cf(n)\}$$

   Prove or disprove (in structured proof form) the following claims:

   (a) Suppose $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$. Then $g \in O(f) \Rightarrow g(n^3) \notin O(f)$

   (b) Suppose $f, g, h : \mathbb{N} \to \mathbb{R}^{\geq 0}$. Then $(g \in O(f) \wedge h \in O(f)) \Rightarrow \max(g, h) \in O(f)$.

   (c) Suppose $f, f', g, g' : \mathbb{N} \to \mathbb{R}^{\geq 0}$, and $f \circ g(n) = f(g(n))$, $f' \circ g'(n) = f'(g'(n))$. Then $(f \in O(f') \wedge g \in O(g')) \Rightarrow f \circ g \in O(f' \circ g')$.

   (d) Suppose $f, g, h : \mathbb{N} \to \mathbb{R}^{\geq 0}$ and $gh(n) = g(n)h(n)$. Then $(g \in O(f) \wedge h \in O(f)) \Rightarrow gh \in O(f)$.

   (e) Suppose $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$ and $f'(n) = \lfloor f(n) \rfloor$, $g'(n) = \lfloor g(n) \rfloor$. Then $g \in O(f) \to g' \in O(f')$.