CSC165, Summer 2005, Assignment 2

Due: Thursday July 14th, 10 am

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Instructions

Please work on all questions. Turn in the outline and structure of a solution, even if you cannot provide every step, and we will try to assign some part marks. However, if there is any question you cannot see how to even begin, leave it blank and you will receive 20% of the marks for that question.

Be sure to give full credit to any sources you consult (other than course notes, TAs, and the instructor) in preparing this problem set. If you try to pass off somebody else's work as your own for credit, you are committing an academic offense, and that can entail serious consequences. Any ideas that you do not attribute to someone else are assumed to be the ideas of the author(s) listed below, and will be evaluated for grading.

Write your name(s) and student number(s) (maximum of two names and two student numbers) in the space below.

Name ______________________________

Student # ______________________________

Name ______________________________

Student # ______________________________

1. Let $\mathbb{N}$ be the natural numbers $\{0, 1, 2, \ldots\}$, $\mathbb{Z}$ be the integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$, and $\mathbb{R}$ be the real numbers. For $x \in \mathbb{R}$, define $r(x)$ as: $\exists m \in \mathbb{N}, \exists n \in \mathbb{N}, (n > 0) \wedge (x = m/n)$. You may assume $\neg r(\sqrt{2})$.

   Using our structured proof form, prove or disprove the following:

   (a) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (r(x) \wedge r(y)) \Rightarrow r(x + y)$.

   (b) The converse of (a)

   (c) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (r(x) \wedge r(y)) \Rightarrow r(xy)$.

   (d) The converse of (c)

2. For $x \in \mathbb{R}$, define $|x|$ by

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

   Using our structured proof form, prove or disprove the following. You may assume that if $t > 0$, then $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x > y \Rightarrow tx > ty$.

   (a) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, |x||y| = |xy|$.

   (b) $\forall x_1 \in \mathbb{R}, \forall x_2 \in \mathbb{R}, \forall y_1 \in \mathbb{R}, \forall y_2 \in \mathbb{R}, (|x_1| > |x_2| \wedge |y_1| > |y_2|) \Rightarrow |x_1 y_1| > |x_2 y_2|$.

3. Let $\mathbb{R}^+$ be the set of positive real numbers. Use our structured proof form to prove or disprove:

   (a)
$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists \delta \in \mathbb{R}^+, \forall \epsilon \in \mathbb{R}^+, |x - y| < \delta \Rightarrow |x^2 - y^2| < \epsilon.$$

   (b)
$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall \epsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, |x - y| < \delta \Rightarrow |x^2 - y^2| < \epsilon.$$

   (c)
$$\forall \epsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in [-1, 1], \forall y \in [-1, 1], |x - y| < \delta \Rightarrow |x^2 - y^2| < \epsilon.$$

   (d)
$$\forall \epsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, \forall y \in \mathbb{R}, |x - y| < \delta \Rightarrow |x^2 - y^2| < \epsilon.$$

4. Suppose f and g are functions from $\mathbb{R}$ onto $\mathbb{R}$. Consider the following statements:

   S1 $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (f(x) = f(y)) \Rightarrow (x = y)$.

   S2 $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (g(x) = g(y)) \Rightarrow (x = y)$.

   S3 $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (g(f(x)) = g(f(y))) \Rightarrow (x = y)$.

   Does (S1$\wedge$S2) imply S3? Prove your claim.