CSC165, Summer 2005, Assignment 2

Due: Thursday June 23rd, 10 am

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Instructions

Please work on all questions. Turn in the outline and structure of a solution, even if you cannot provide every step, and we will try to assign some part marks. However, if there is any question you cannot see how to even begin, leave it blank you will receive 20% of the marks for that question.

Be sure to give full credit to any sources you consult (other than course notes, TAs, and the instructor) in preparing this problem set. If you try to pass off somebody else's work as your own for credit, you are committing an academic offense, and that can entail serious consequences. Any ideas that you do not attribute to someone else are assumed to be the ideas of the author(s) listed below, and will be evaluated for grading.

Write your name(s) and student number(s) (maximum of two names and two student numbers) in the space below.

Name ______________________________

Student # ______________________________

Name ______________________________

Student # ______________________________

1. For $x, y, z \in \mathbb{R}$ define:

   - $d(x, y, z)$ means $x/y = z$.
   - $s(x, y, z)$ means $x + y = z$.
   - $eq(x, y)$ means $x = y$.
   - $g(x, y)$ means $x > y$.
   - $\mathbb{N}(x)$ means $x \in \mathbb{N}$, where $\mathbb{N} = \{0, 1, 2, \ldots\}$ (the natural numbers).

   Express each of the following symbolic sentences in English. If the sentence is false, provide a counterexample. If it's true, explain why.

   (a) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, s(x, y, z)$.

   (b) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, \mathbb{N}(x) \wedge \mathbb{N}(y) \wedge s(x, y, z) \Rightarrow \mathbb{N}(z)$.

   (c) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, d(x, y, z)$.

   (d) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, s(x, y, x)$.

   (e) $\forall y \in \mathbb{R}, \forall x \in \mathbb{R}, s(x, y, x) \Rightarrow eq(y, 0)$.

   (f) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, d(x, y, x) \Rightarrow eq(y, 1)$.

   (g) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, s(x, y, 0)$.

   (h) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, s(x, y, 0)$.

   (i) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, g(x, y) \wedge d(z, x, y) \Rightarrow g(z, y)$.

2. Using the predicates defined in the previous question, and $\mathbb{N} = \{0, 1, 2, \ldots, \}$:

   (a) Translate the following into English:

   $$\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, \forall z \in \mathbb{N}, d(x, 3, y) \wedge d(z, x, x) \Rightarrow \exists w \in \mathbb{N}, d(z, 3, w)$$

   (b) If the sentence you translated above is false, provide a counterexample. Otherwise, write a direct proof of the implication.

3. Consider the following java method (if you have ANY questions about java, please feel free to ask your TA or instructor):

```
public static boolean longPredicate(boolean p, boolean q) {
 return (
        ((!p || q) && (p || !q) && !q) ||
        (!(p && !q) && !(!p && q) && p)
          );
}
```

   (a) Make a literal translation of the conditional expression being returned into precise symbolic notation, using p, q, $\wedge$, $\vee$, and $\neg$, but without using $\Rightarrow$ or $\Leftrightarrow$. p and q should occur as many times in your expression as they do in the java conditional expression.

   (b) Simplify your expression from the previous part, allowing yourself to use $\Rightarrow$ or $\Leftrightarrow$ where appropriate. Use p and q no more than twice each. Explain the steps you took in simplifying the expression.

   (c) Write the negation of the previous part in precise symbolic notation, and explain its meaning in English.

4. Define $P(x)$ as "x is pedantic," $Q(x)$ as "x is a quibbler," $R(x)$ as "x is redundant," and S is the domain of scholars. For each sentence below:

   - Write the negation of the sentence in precise symbolic notation, moving the negation symbol $\neg$ as close as possible to the predicates P, Q or R as possible.
   - Draw a Venn diagram showing P, Q, R and S for which the original sentence is false.

   Here are the sentences:

   (a) Any redundant scholar is a quibbler only if he/she is pedantic.

   (b) Some scholar is not redundant if he/she is neither a quibbler nor non-pedantic.

   (c) All redundant and pedantic scholars must be quibblers.

   (d) Some scholar is either not both pedantic and a quibbler, or is redundant.

   (e) Whenever any scholar is a quibbler, there is a scholar who is redundant and pedantic.

5. In the previous question, is (a) equivalent to the negation of (b)? Either provide a counter-example or show they are equivalent.