

CSC165, Summer 2005, Assignment 1

Sample solution

Danny Heap

1. PREREQUISITES

The following fragments are from the U of T Arts & Science Calendar, specifying the prerequisites for CSC165:

CSC107H1/CSC108H1/(CSC148H1/CSC150H1 taken concurrently); U Advanced Functions and Introductory Calculus, and one of U Geometry and Discrete Mathematics or U Mathematics of Data Management OR OAC Calculus and one of Algebra & Geometry or Finite Mathematics

⋮

Explanation of Symbols The comma (,) the semi-colon(;) the ampersand (&) and the plus sign (+) all mean “AND”. The slash (/) means “OR”.

Devise symbolic notation for each of the predicates connecting a single course to a student. For example, $CSC107(x)$ may represent “ x has completed $CSC107H1$.” Using the operators \wedge and \vee , combine your predicates into three DIFFERENT predicates, each of which is CONSISTENT with the given English description of the prerequisites. By DIFFERENT I mean the predicates specify different sets of students, and by CONSISTENT I mean that there is a reasonable interpretation of the English description that has the same meaning as your predicate. Justify your answers.

SAMPLE SOLUTION: There is a great deal of room for finding ambiguity in the A&S Calendar fragments. Define the following:

- D is the domain of registered U of T students
- $CSC107(x)$ means x completed CSC107
- $CSC108(x)$ means x completed CSC108
- $CSC148(x)$ means x is taking CSC148 concurrently
- $CSC150(x)$ means x is taking CSC150 concurrently
- $AFIC(x)$ means x completed Advanced Functions and Introductory Calculus
- $GDM(x)$ means x completed Geometry and Discrete Mathematics
- $MDM(x)$ means x completed Mathematics of Data Management
- $OACC(x)$ means x completed OAC Calculus
- $AG(x)$ means x completed Algebra and Geometry
- $FM(x)$ means x completed Finite Mathematics

In all predicates I assume that the parentheses are used to indicate that either CSC148 or CSC150 may be taken concurrently.

My first predicate indicates what I THINK the authors of the Calendar fragment actually meant. I am assuming that “OR” is evaluated before “AND,” and that “one of,” means at least one of. Given this interpretation, student $x \in D$ satisfies the prerequisites if:

$$\begin{aligned} (CSC107(x) \vee CSC108(x) \vee CSC148(x) \vee CSC150(x)) &\wedge AFIC(x) \\ &\wedge (GDM(x) \vee MDM(x) \vee OACC(x)) \\ &\wedge (AG(x) \vee FM(x)) \end{aligned}$$

My second predicate has the same assumptions as the first predicate, except I assume “one of” means exactly one of, so the following “ORs” behave like an exclusive “ORs.”

$$\begin{aligned} (CSC107(x) \vee CSC108(x) \vee CSC148(x) \vee CSC150(x)) &\wedge AFIC(x) \\ &\wedge (GDM(x) \vee MDM(x) \vee OACC(x)) \\ &\wedge (\neg GDM(x) \vee \neg MDM(x)) \\ &\wedge (\neg GDM(x) \vee \neg OACC(x)) \\ &\wedge (\neg MDM(x) \vee \neg OACC(x)) \\ &\wedge (AG(x) \vee FM(x)) \end{aligned}$$

My third predicate assumes that “AND” is evaluated before “OR” except where “AND” is followed by “one of.”

$$\begin{aligned} CSC107(x) \vee CSC108(x) \vee CSC148(x) \\ \vee (CSC150(x) \wedge AFIC(x) \wedge \\ \wedge (GDM(x) \vee MDM(x) \vee OACC(x)) \wedge (AG(x) \vee FM(x))) \end{aligned}$$

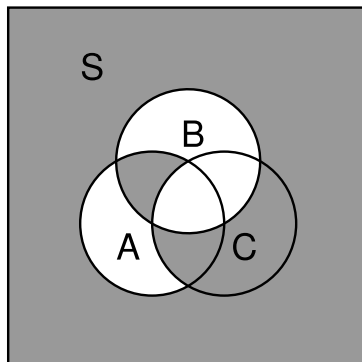
A student who has completed CSC107 and none of the other courses satisfies the third predicate and not the other two. A student who has taken CSC107, AFIC, GDM, MDM, and AG satisfies the first predicate and not the second. Thus the three predicates are different.

2. THE VENN OF STATEMENTS

Let S represent the set of stories, and

- Let $A(s)$ represent “ s is apocryphal.”
- Let $B(s)$ represent “ s is blasphemous.”
- Let $C(s)$ represent “ s is convoluted.”

Consider this Venn diagram, where shaded regions are exactly those containing one or more stories.



Which of the following are true, which are false? Justify your answers using the Venn diagram.

(a) $\forall s \in S, A(s) \Rightarrow B(s)$.

False, since the shading in $(A \cap C) \setminus B$ indicates there is a counter-example.

(b) Every blasphemous story is apocryphal.

True, since there is no shading in $B \setminus A$ to indicate a counterexample.

(c) $\exists s \in S, C(s) \Rightarrow A(s)$.

True. There are 3 shaded regions outside of $C \setminus A$ that provide examples.

(d) Some story is both apocryphal and convoluted only if it is blasphemous.

True. There are 3 shaded regions outside of $(A \cap C) \setminus B$ that provide examples.

(e) $\exists s \in S, B(s) \Rightarrow C(s)$.

True. The three shaded regions outside of $B \setminus C$ provide examples.

(f) Any story that is both apocryphal and blasphemous must be convoluted.

False. The shaded region $(A \cap B) \setminus C$ provides a counterexample.

3. VARIOUS VENNS

For some domain D , consider the statement:

$$\forall x \in D, E(x) \Rightarrow (F(x) \Rightarrow G(x)).$$

Draw three unshaded Venn diagrams with domain D and intersecting sets E , F , and G that create 8 regions analogous to the diagram in question 2. Now shade each diagram to indicate which regions are non-empty, so that no diagram contradicts the above statement. In addition, each pair of diagrams must be distinct, having at least one region that's shaded in one and unshaded in another. Justify your diagrams.

SAMPLE SOLUTION: The only way to make the statement false is for $E(x)$ to be true while $(F(x) \Rightarrow G(x))$ is false, in other words $E(x)$ and $F(x)$ are true while $G(x)$ is false. So the only Venn diagram that contradicts the statement is one in which the region $(E \cap F) \setminus G$ is shaded. Any diagram that leaves $(E \cap F) \setminus G$ blank doesn't contradict the statement, and there are 2^7 ways to shade the remaining 7 regions (so I'm not going to draw them all here). You need 3 distinct diagrams from those 2^7 .

4. VIRTUE REWARDED

The following piece of propaganda undertakes to show the correlation between the number of hours per week (including lectures and tutorials) spent on CSC165 and final mark.

student	mark	hours/week
<i>A</i>	65	4
<i>B</i>	70	5
<i>C</i>	75	6
<i>D</i>	80	7
<i>E</i>	85	8
<i>F</i>	90	3

If our domain D is $\{A, B, C, D, E, F\}$, then which of the following conclusions are justified or not justified? Explain your answers.

- (a) Any student who spends at least 7 hours per week on the course earns a mark of at least 80.
True, since the only students who spend at least 7 hours per week on the course are D and E , and they earn at least 80.
- (b) All students who earn a mark of at least 80 study at least 7 hours per week.
False, since F is a counterexample, earning at least 80 but spending 3 hours.
- (c) All students who study at least 9 hours per week earn a mark of at least 90.
Vacuously true, since there are no students who study at least 9 hours per week, and hence no counterexamples.
- (d) All students who study less than 3 hours per week earn a mark of at least 95.
Vacuously true, since there are no students who study less than 3 hours per week, and hence no counterexamples.
- (e) Not all students who study less than 2 hours per week earn a mark of less than 65.
False, since it negates a vacuously true statement. There are no students who study less than 2 hours per week, hence no counterexamples who earn a mark of at least 65.