# One-Hot Encoding

$$\begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}$$

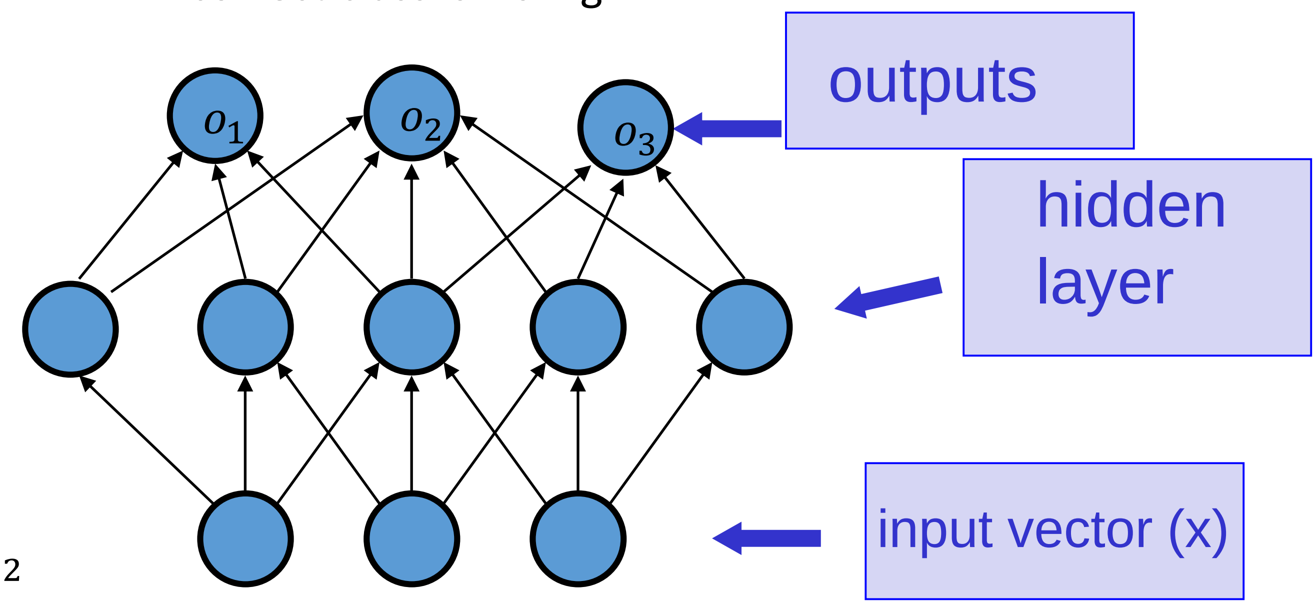
$$\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}$$

# One-Hot Encoding

- Data: $\left(x^{(1)}, y^{(1)}\right), \left(x^{(2)}, y^{(2)}\right), \ldots \left(x^{(n)}, y^{(n)}\right)$
- E.g., $y^{(i)} \in \{\text{"person"}, \text{"hamster"}, \text{"capybara"}\}$
- Encode as $y^{(i)} \in \{1, 2, 3\}$?
  - Shouldn't be running something like linear regression, since "hamster" is not really the average of "person" and "capybara," so things are not likely to work well
    (Explanation on the board)
- Solution: one-hot encoding
  - "person" => [1, 0, 0]
  - "hamster" => [0, 1, 0]
  - "capybara" => [0, 0, 1]

# Multilayer Neural Network for Classification

$o_i$ is large if the probability that the correct class is *i* is high




A possible cost function:

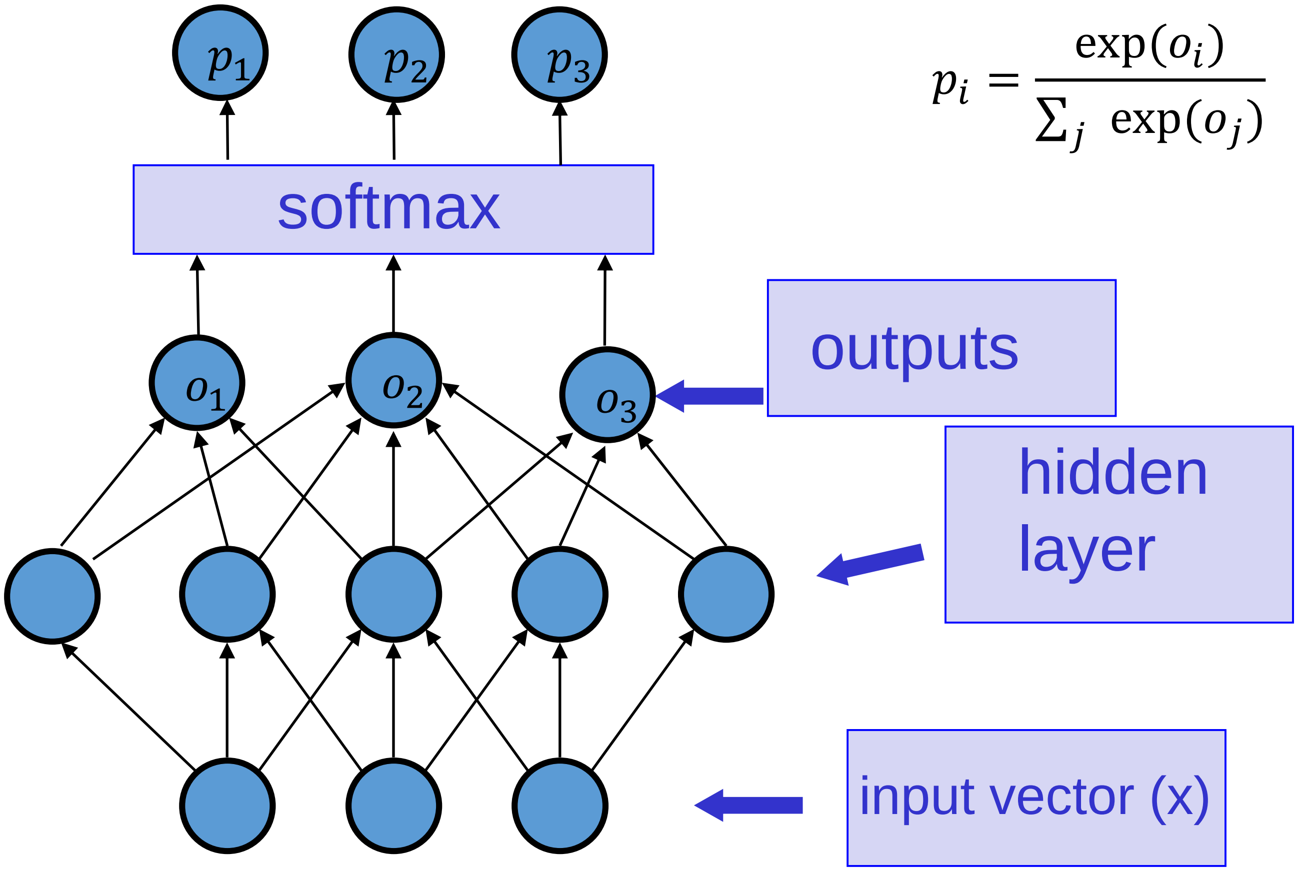
$$\sum_{i=1}^{m} \left(o^{(i)} - y^{(i)}\right)^2$$

$y^{(i)'}s$ encoded using one-hot encoding

# Softmax

- Want to estimate the probability $P(y = y'|x, \theta)$
  - $\theta$: network parameters

$$p_i = \frac{\exp(o_i)}{\sum_j \exp(o_j)}$$



# Softmax

- $p_i = \frac{\exp(o_i)}{\sum_j \exp(o_j)}$ can be thought of as probabilities
  - $0 < p_i < 1$
  - $\sum_j p_j = 1$
  - This is a generalization of logistic regression
    - (For two outputs, $p_1 = \frac{\exp(o_1)}{\exp(o_1)+\exp(o_2)} = \frac{1}{1+\exp(o_2-o_1)}$ )

# Cost Function: $-\sum_j y_j log p_j$

- Negative log-probability of the correct answer
  - The probability of getting the answer correct if we are guessing according to Prob(guessing i)=$p_i$ is

    $p_i$
  - If the right answer is $i$,

    $y_i = 1$ and $y_j = 0$ for $i \neq j$
  - So the probability of getting the answer correct is

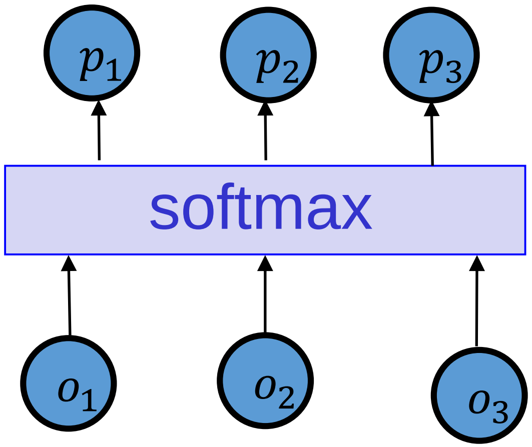
$$\sum_j y_j p_j$$

The negative log-probability of getting the correct answer is then
$-\sum_j y_j log p_j$ for $p_j$ equal to the probability of class $j$ according to the Neural Network

# Cost Function Gradient

$$p_i = \frac{e^{o_i}}{\sum_j e^{o_j}}$$

$$\frac{\partial p_i}{\partial o_i} = p_i\,(1 - p_i)$$




$$C = -\sum_j y_j \log p_j$$

$$\frac{\partial C}{\partial o_i} = \sum_j \frac{\partial C}{\partial p_j}\frac{\partial p_j}{\partial o_i} = p_i - y_i$$