Suppose we have a function \( f \) of two variables. At the point \((x_0, y_0)\), what is the vector that points in the direction of steepest ascent?

As discussed in lecture, in order to increase \( f \) the most in the neighborhood of \((x_0, y_0)\), we need to move to

\[
(x_0 + \alpha \frac{\partial f}{\partial x}(x_0, y_0) + \alpha \frac{\partial f}{\partial y}(x_0, y_0))
\]

for some \( \alpha \).

But where do you move along the \( z \) axis? Near \((x_0, y_0)\), moving \( x \) by \( h \) moves \( f(x, y) \) by \( h \frac{\partial f}{\partial x}(x_0, y_0) \), and moving \( y \) by \( h \) moves \( f(x, y) \) by \( h \frac{\partial f}{\partial y}(x_0, y_0) \). We are doing both of those simultaneously, and the result is moving \( f \) by

\[
\alpha(\frac{\partial f}{\partial x}(x_0, y_0)^2 + \frac{\partial f}{\partial x}(x_0, y_0)^2)
\]

The vector is therefore

\[
(\frac{\partial f}{\partial x}(x_0, y_0), \frac{\partial f}{\partial y}(x_0, y_0), \frac{\partial f}{\partial x}(x_0, y_0)^2 + \frac{\partial f}{\partial x}(x_0, y_0)^2)^T.
\]