PCA, Eigenfaces, and Face Detection

Salvador Dalí, “Galatea of the Spheres”

Many slides from
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CSC320: Introduction to Visual Computing
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What makes face detection hard?

*Face detection: given an image, find the coordinates of the faces

Variation in appearance: can’t match a single face template and expect it to work.
(Note: it doesn’t work that great for eyes either)
What makes face detection hard?

Lighting
What makes face detection hard?

Occlusion
What makes face recognition hard?

Viewpoint
Face detection

• Do these images contain faces? Where?
Simple Idea for Face Detection

1. Treat each window in the image like a vector

2. Test whether $x$ matches some $y_j$ in the database

SSD: $(y_j - x)^2$

Cross-correlation: $y_j \cdot x$

NCC, zero-mean NCC…
The space of all face images

- When viewed as vectors of pixel values, face images are extremely high-dimensional
  - 100x100 image = 10,000 dimensions
  - Slow and lots of storage
- But very few 10,000-dimensional vectors are valid face images
- We want to effectively model the subspace of face images
The space of all face images

- Eigenface idea: construct a low-dimensional linear subspace that contains most of the face images possible (possibly with small errors)

- Here: a 1D subspace arguably suffices
The space of faces

An image is a point in a high dimensional space
- An $W \times H$ intensity image is a point in $R^{WH}$
- We can define vectors in this space as we did in the 2D case
Reconstruction

- For a subspace with the orthonormal basis of size $k$ $V_k = \{v_0, v_1, v_2, \ldots v_k\}$, the best reconstruction of $x$ in that subspace is:
  \[
  \hat{x}_k = (x \cdot v_0)v_0 + (x \cdot v_1)v_1 + \cdots + (x \cdot v_k)v_k
  \]
  - If $x$ is in the span of $V_k$, this is an exact reconstruction
  - If not, this is the projection of $x$ on $V$

- Squared reconstruction error: $(\hat{x}_k - x)^2$
Reconstruction cont’d

• $\hat{x}_k = (x \cdot v_0)v_0 + (x \cdot v_1)v_1 + \cdots + (x \cdot v_k)v_k$

• Note: in $(x \cdot v_0)v_0$,
  – $(x \cdot v_0)$ is a measure of how similar $x$ is to $v_0$
  – The more similar $x$ is to $v_0$, the larger the contribution from $v_0$ is to the sum
Representation and reconstruction

- Face $\mathbf{x}$ in “face space” coordinates:

$$\mathbf{x} \rightarrow \begin{bmatrix} u_1^T (\mathbf{x} - \mu) , \ldots , u_k^T (\mathbf{x} - \mu) \end{bmatrix}$$

$$= w_1 , \ldots , w_k$$

- Reconstruction:

$$\hat{\mathbf{x}} = \mu + w_1 u_1 + w_2 u_2 + w_3 u_3 + w_4 u_4 + \ldots$$
After computing eigenfaces using 400 face images from ORL face database.
Principal Component Analysis

• Suppose the columns of a matrix $X_{N \times K}$ are the datapoints (N is the size of each image, K is the size of the dataset), and we would like to obtain an orthonormal basis of size k that produces the smallest sum of squared reconstruction errors for all the columns of $X - \bar{X}$

  $\bar{X}$ is the average column of X

• Answer: the basis we are looking for is the k eigenvectors of $(X - \bar{X})(X - \bar{X})^T$ that correspond to the k largest eigenvalues
PCA – cont’d

• \((X - \bar{X})(X - \bar{X})^T\) is called the covariance matrix

• If \(x\) is the datapoint (obtained after subtracting the mean), and \(V\) an orthonormal basis, \(V^T x\) is a column of the dot products of \(x\) and the elements of \(x\)

• So the reconstruction for the centered \(x\) is \(\hat{x} = V(V^T x)\)

• PCA is the procedure of obtaining the \(k\) eigenvectors \(V_k\)
NOTE: centering

• If the image $x$ is *not centred* (i.e., $\bar{X}$ was not subtracted), the reconstruction is:
  \[
  \hat{x} = \bar{X} + V(V^T(x - \bar{X}))
  \]
Proof that PCA produces the best reconstruction

- (Fairly easy calculus – look it up, or we can talk in office hours, or possibly we’ll do it next week)
Obtaining the Principal Components

• $XX^T$ can be huge
• There are tricks to still compute the EVs
PCA as dimensionality reduction

The set of faces is a “subspace” of the set of images

- Suppose it is K dimensional
- We can find the best subspace using PCA
- This is like fitting a “hyper-plane” to the set of faces
  - spanned by vectors \( \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K \)
  - any face \( \mathbf{x} \approx \bar{\mathbf{x}} + a_1 \mathbf{v}_1 + a_2 \mathbf{v}_2 + \ldots + a_k \mathbf{v}_k \)
Eigenfaces example

Mean: $\mu$

Top eigenvectors: $u_1, \ldots, u_k$
Another Eigenface set
Linear subspaces

Convert $x$ into $v_1, v_2$ coordinates:

$$x \rightarrow ((x - \bar{x}) \cdot v_1, (x - \bar{x}) \cdot v_2)$$

What does the $v_2$ coordinate measure?
- distance to line
- use it for classification—near 0 for orange pts

What does the $v_1$ coordinate measure?
- position along line
- use it to specify which orange point it is
Dimensionality reduction

We can represent the orange points with *only* their $v_1$ coordinates
- since $v_2$ coordinates are all essentially 0

This makes it much cheaper to store and compare points

A bigger deal for higher dimensional problems
Another Interpretation of PCA

The eigenvectors of the covariance matrix define a new coordinate system

- eigenvector with largest eigenvalue captures the most variation among training vectors $\mathbf{x}$
- eigenvector with smallest eigenvalue has least variation
- The eigenvectors are known as principal components
Data Compression using PCA

• For each data point $x$, store $V_k^T x$ (a $k$-dimensional vector). The reconstruction error would be the smallest for a set of $k$ numbers.
Face Detection using PCA

- For each (centered) window $x$ and for a set of principal components $V$, compute the Euclidean distance $|VV^T x - x|$

- That is the distance between the reconstruction of $x$ and $x$. The reconstruction of $x$ is similar to $x$ if $x$ lies in the face subspace
  - Note: the reconstruction is *always* in the face subspace
Issues: dimensionality

What if your space isn’t *flat*?

- PCA may not help

Nonlinear methods
LLE, MDS, etc.
Moving forward

• Faces are pretty well-behaved
  – Mostly the same basic shape
  – Lie close to a low-dimensional subspace

• Not all objects are as nice
Different appearance, similar parts