# Understanding How Neural Networks See

SML201: Introduction to Data Science, Spring 2019

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## Recent successes of neural networks

- Can recognize what object is in the a photo
- Can tell bad Go positions/shapes from good Go positions
- Cen tell a self-driving car where to go
- Can decide on what key to press to win at a video game by looking at the screen

## About this lecture

- A *very* brief introduction to artificial neural networks (ANNs)
  - Why and how ANNs work
- A *very* brief introduction to Explainable AI
  - Understanding how “black box” models work

## “Review:” Supervised Machine Learning

› Training set:

- Training example 1: $\mathrm{x}^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, \dots, x_m^{(1)}\right)$ output: $y^{(1)}$
- Training example 2: $\mathrm{x}^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, \dots, x_m^{(2)}\right)$ output: $y^{(2)}$

…

- Training example N: $\mathrm{x}^{(\mathrm{N})} = \left(x_1^{(N)}, x_2^{(N)}, \dots, x_m^{(N)}\right)$ output: $y^{(N)}$

› Test set:

- Test Example 1: $\mathrm{x}^{(N+1)} = \left(x_1^{(N+1)}, x_2^{(N+1)}, \dots, x_m^{(N+1)}\right)$ output: $y^{(N+1)}$
- Test Example 2: $\mathrm{x}^{(N+2)} = \left(x_1^{(N+2)}, x_2^{(N+2)}, \dots, x_m^{(N+2)}\right)$ output: $y^{(N+2)}$
- …
- Test Example K: $\mathrm{x}^{(N+K)} = \left(x_1^{(N+K)}, x_2^{(N+K)}, \dots, x_m^{(N+K)}\right)$ output: $y^{(N+K)}$

- Goal: Find a $\theta$ such that $h_\theta\left(x^{(i)}\right) \approx y^{(i)}$ for $i \in 1, \dots, N$
- Hope: $h_\theta\left(x^{(i)}\right) \approx y^{(i)}$ for any $i$
- For new input $x$, predict $h_\theta(x)$

Shotgun debugging

## Machine Learning vs. Intro to Programming

› Programming ***done badly***

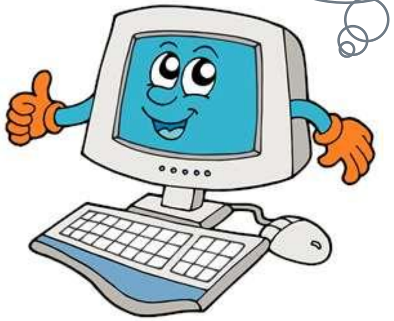
```
CountryMaxIncome <- function(gap):
  return(min(gap$gdpPercap)

> CountryMaxIncome(gapminder)
10
```

Change the `min` to `max`?

› Machine Learning ***done right***

>>> $h_{(0,1.2,0.1)}$([0, 0])
[0, 0]
>>> $h_{(0,1.2,0.1)}$([1, 2])
[1.3, 2.8]

$$h_{(\theta_1,\theta_2,\theta_3)}(x) = \theta_1 + \theta_2 x + \theta_3 x^2$$

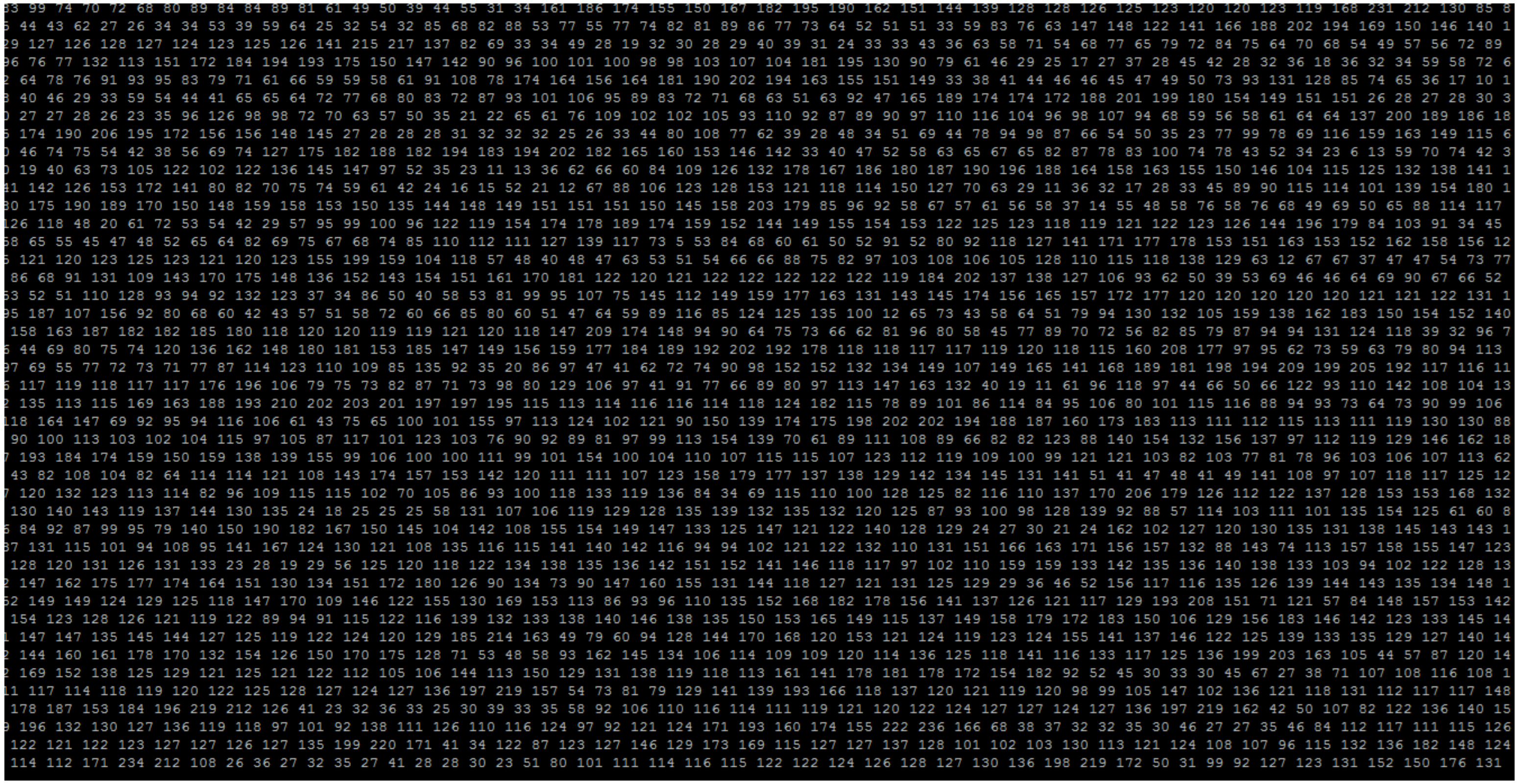


Change $\theta_2$ to 1.3?

Machine learning (kind of)

## Sample ML task: Recognizing Justin Bieber

## What Justin Bieber looks like to a computer



```
33 99 74 70 72 68 80 89 84 84 89 81 61 49 50 39 44 55 31 34 161 186 174 155 150 167 182 195 190 162 151 144 139 128 128 126 125 123 120 120 123 119 168 231 212 130 85 8
5 44 43 62 27 26 34 34 53 39 59 64 25 32 54 32 85 68 82 88 53 77 55 77 74 82 81 89 86 77 73 64 52 51 51 33 59 83 76 63 147 148 122 141 166 188 202 194 169 150 146 140 1
29 127 126 128 127 124 123 125 126 141 215 217 137 82 69 33 34 49 28 19 32 30 28 29 40 39 31 24 33 33 43 36 63 58 71 54 68 77 65 79 72 84 75 64 70 68 54 49 57 56 72 89
96 76 77 132 113 151 172 184 194 193 175 150 147 142 90 96 100 101 100 98 98 103 107 104 181 195 130 90 79 61 46 29 25 17 27 37 28 45 42 28 32 36 18 36 32 34 59 58 72 6
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3 40 46 29 33 59 54 44 41 65 65 64 72 77 68 80 83 72 87 93 101 106 95 89 83 72 71 68 63 51 63 92 47 165 189 174 174 172 188 201 199 180 154 149 151 151 26 28 27 28 30 3
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0 46 74 75 54 42 38 56 69 74 127 175 182 188 182 194 183 194 202 182 165 160 153 146 142 33 40 47 52 58 63 65 67 65 82 87 78 83 100 74 78 43 52 34 23 6 13 59 70 74 42 3
0 19 40 63 73 105 122 102 122 136 145 147 97 52 35 23 11 13 36 62 66 60 84 109 126 132 178 167 186 180 187 190 196 188 164 158 163 155 150 146 104 115 125 132 138 141 1
41 142 126 153 172 141 80 82 70 75 74 59 61 42 24 16 15 52 21 12 67 88 106 123 128 153 121 118 114 150 127 70 63 29 11 36 32 17 28 33 45 89 90 115 114 101 139 154 180 1
30 175 190 189 170 150 148 159 158 153 150 135 144 148 149 151 151 151 150 145 158 203 179 85 96 92 58 67 57 61 56 58 37 14 55 48 58 76 58 76 68 49 69 50 65 88 114 117
126 118 48 20 61 72 53 54 42 29 57 95 99 100 96 122 119 154 174 178 189 174 159 152 144 149 155 154 153 122 125 123 118 119 121 122 123 126 144 196 179 84 103 91 34 45
58 65 55 45 47 48 52 65 64 82 69 75 67 68 74 85 110 112 111 127 139 117 73 5 53 84 68 60 61 50 52 91 52 80 92 118 127 141 171 177 178 153 151 163 153 152 162 158 156 12
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 158 163 187 182 182 185 180 118 120 120 119 119 121 120 118 147 209 174 148 94 90 64 75 73 66 62 81 96 80 58 45 77 89 70 72 56 82 85 79 87 94 94 131 124 118 39 32 96 7
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2 147 162 175 177 174 164 151 130 134 151 172 180 126 90 134 73 90 147 160 155 131 144 118 127 121 131 125 129 29 36 46 52 156 117 116 135 126 139 144 143 135 134 148 1
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 154 123 128 126 121 119 122 89 94 91 115 122 116 139 132 133 138 140 146 138 135 150 153 165 149 115 137 149 158 179 172 183 150 106 129 156 183 146 142 123 133 145 14
1 147 147 135 145 144 127 125 119 122 124 120 129 185 214 163 49 79 60 94 128 144 170 168 120 153 121 124 119 123 124 155 141 137 146 122 125 139 133 135 129 127 140 14
2 144 160 161 178 170 132 154 126 150 170 175 128 71 53 48 58 93 162 145 134 106 114 109 109 120 114 136 125 118 141 116 133 117 125 136 199 203 163 105 44 57 87 120 14
2 169 152 138 125 129 121 125 121 122 112 105 106 144 113 150 129 131 138 119 118 113 161 141 178 181 178 172 154 182 92 52 45 30 33 30 45 67 27 38 71 107 108 116 108 1
11 117 114 118 119 120 122 125 128 127 124 127 136 197 219 157 54 73 81 79 129 141 139 193 166 118 137 120 121 119 120 98 99 105 147 102 136 121 118 131 112 117 117 148
 178 187 153 184 196 219 212 126 41 23 32 36 33 25 30 39 33 35 58 92 106 110 116 114 111 119 121 120 122 124 127 127 124 127 136 197 219 162 42 50 107 82 122 136 140 15
9 196 132 130 127 136 119 118 97 101 92 138 111 126 110 116 124 97 92 121 124 171 193 160 174 155 222 236 166 68 38 37 32 32 35 30 46 27 27 35 46 84 112 117 111 115 126
 122 121 122 123 127 127 126 127 135 199 220 171 41 34 122 87 123 127 146 129 173 169 115 127 127 137 128 101 102 103 130 113 121 124 108 107 96 115 132 136 182 148 124
 114 112 171 234 212 108 26 36 27 32 35 27 41 28 28 30 23 51 80 101 111 114 116 115 122 122 124 126 128 127 130 136 198 219 172 50 31 99 92 127 123 131 152 150 176 131
```
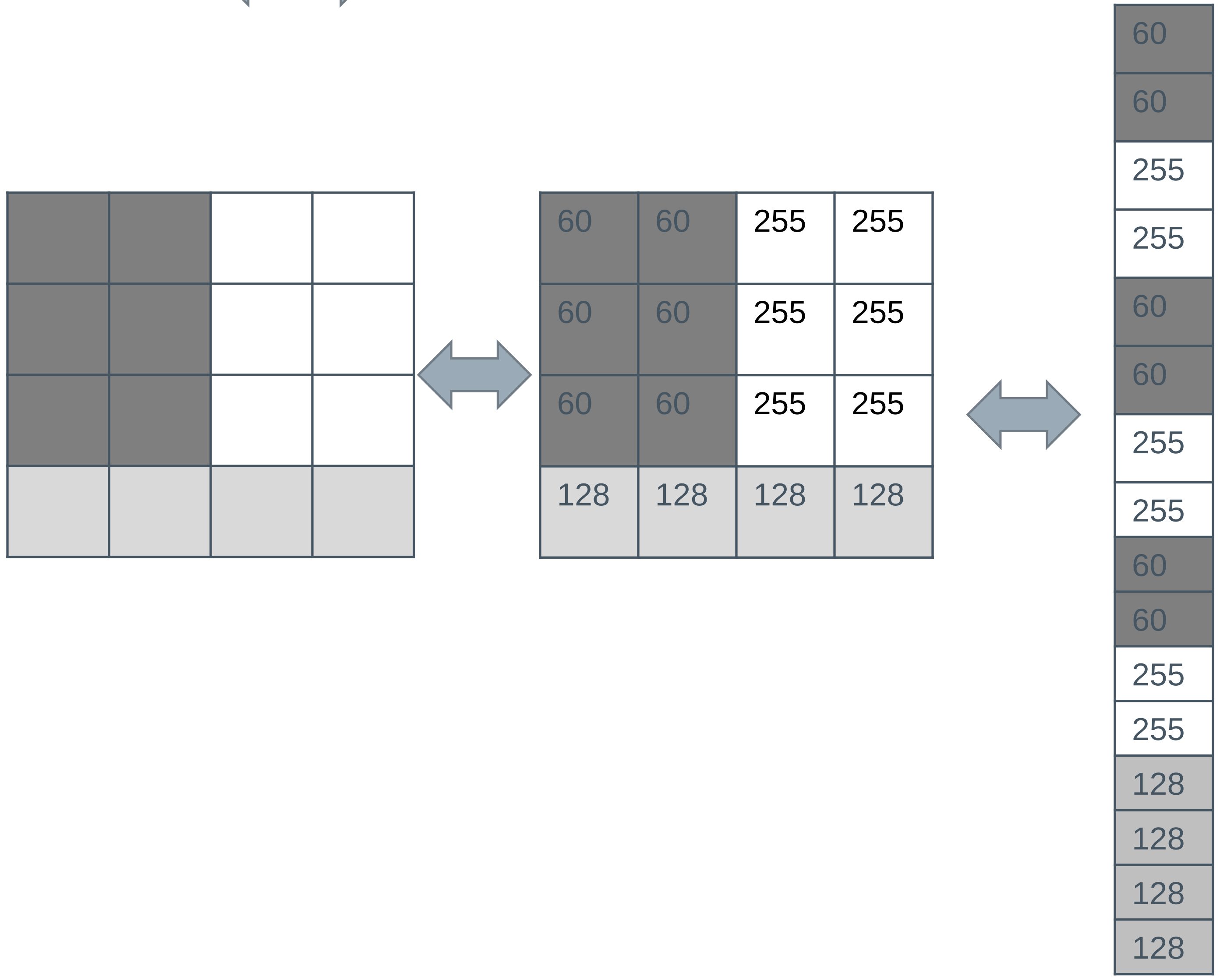

## Images ⟷ Vectors



## The Face Recognition Task

- Training set:
  - $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(N)}, y^{(N)})\}$
    - $x^{(i)}$ is a k-dimensional vector consisting of the intensities of all the pixels in in the i-th photo ($20 \times 20$ photo $\rightarrow x^{(i)}$ is 400-dimensional)
    - $y^{(i)}$ is the *label* (i.e., name)
- Test phase:
  - We have an input vector $x$, and want to assign a label $y$ to it
    - Whose photo is it?

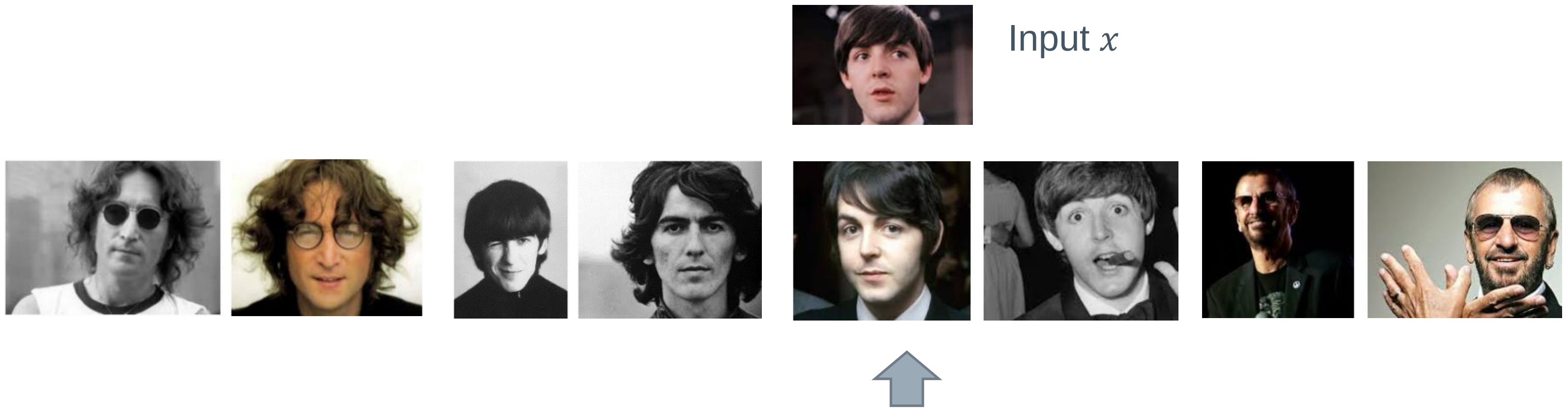
## Face Recognition using 1-Nearest Neighbors (1NN)

– Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(N)}, y^{(N)})\}$

– Input: $x$

– 1-Nearest Neighbor algorithm:

- Find the training photo/vector $x^{(i)}$ that's as "close" as possible to $x$, and output the label $y^{(i)}$



Input $x$

Closest training image to the input $x$

Output: Paul

## Supervised Machine Learning

› Training set:

- Training example 1: $\mathrm{x}^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, \ldots., x_m^{(1)}\right)$ output: $y^{(1)}$
- Training example 2: $\mathrm{x}^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, \ldots., x_m^{(2)}\right)$ output: $y^{(2)}$

…

- Training example N: $\mathrm{x}^{(\mathrm{N})} = \left(x_1^{(N)}, x_2^{(N)}, \ldots., x_m^{(N)}\right)$ output: $y^{(N)}$

› Test set:

- Test Example 1: $\mathrm{x}^{(N+1)} = \left(x_1^{(N+1)}, x_2^{(N+1)}, \ldots., x_m^{(N+1)}\right)$ output: $y^{(N+1)}$
- Test Example 2: $\mathrm{x}^{(N+2)} = \left(x_1^{(N+2)}, x_2^{(N+2)}, \ldots., x_m^{(N+2)}\right)$ output: $y^{(N+2)}$
- …
- Test Example K: $\mathrm{x}^{(N+K)} = \left(x_1^{(N+K)}, x_2^{(N+K)}, \ldots., x_m^{(N+K)}\right)$ output: $y^{(N+K)}$

- Goal: Find a $\theta$ such that $h_\theta\left(x^{(i)}\right) \approx y^{(i)}$ for $i \in 1, \ldots, N$
- Hope: $h_\theta\left(x^{(i)}\right) \approx y^{(i)}$ for any $i$
- For new input $x$, predict $h_\theta(x)$

## Are the two images $a$ and $b$ close?

- Key idea: think of the images as *vectors*
  - Reminder: to turn an image into a vector, simply "flatten" all the pixels into a 1D vector
- Is the distance between the endpoints of vectors $a$ and $b$ small?

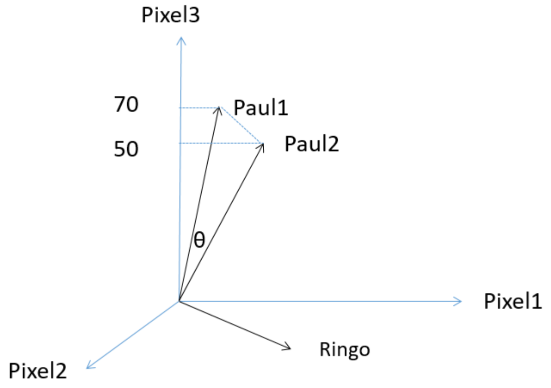
$$|a-b| = \sqrt{\sum_i (a_i - b_i)^2} \text{ small}$$

- Is the cosine of the angle between the vectors $a$ and $b$ large?

By the law of cosines

$$\cos\theta_{ab} = \frac{a \cdot b}{|a||b|} = \frac{\sum_i a_i b_i}{\sqrt{\sum_i a_i^2}\sqrt{\sum_i b_i^2}} \text{ large}$$

- Is $a \cdot b = \sum_i a_i b_i$ large?
  - Assume $|a| \approx |b| \approx const$

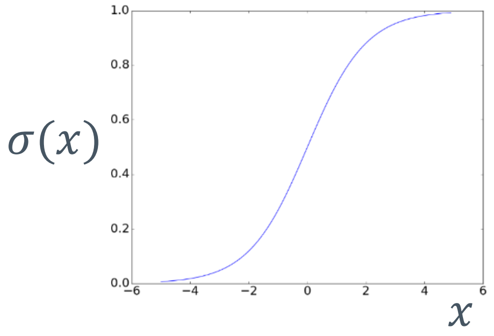


## SML310 Project 3 task

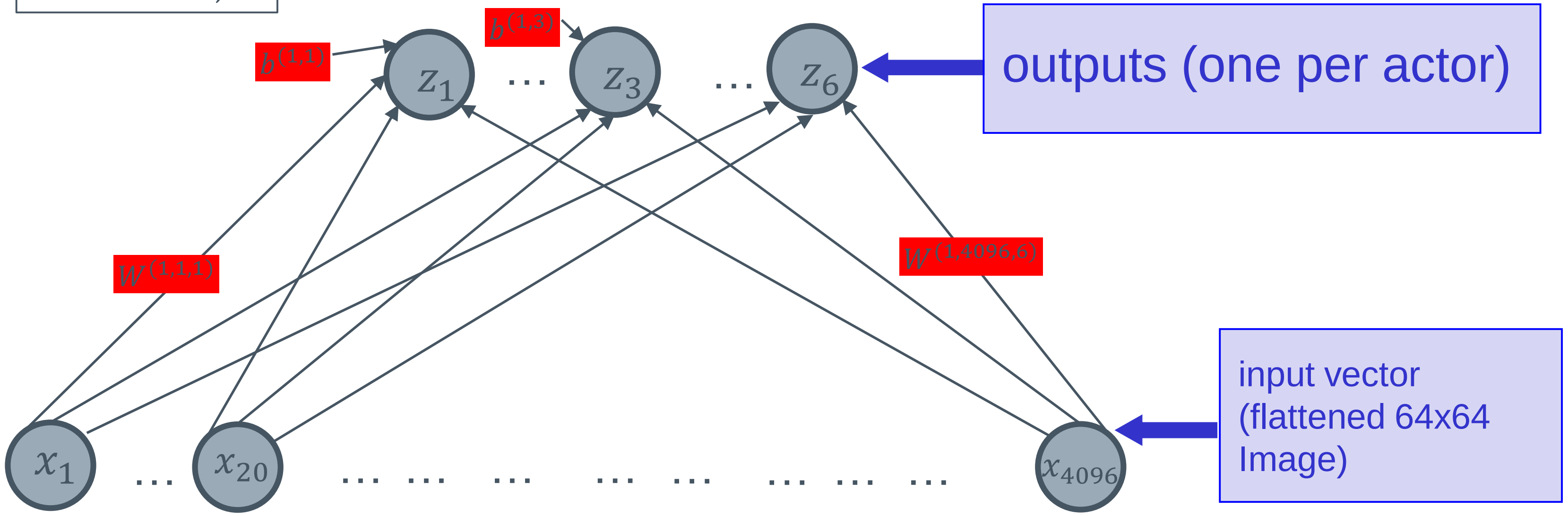
- Training set: 6 actors, with 100 $64 \times 64$ photos of faces for each
- Test set: photos of faces of the same 6 actors
- Want to classify each face as one of ['Fran Drescher', 'America Ferrera', 'Kristin Chenoweth', 'Alec Baldwin', 'Bill Hader', 'Steve Carell']

# The Simplest Possible Neural Network for Face Recognition

$$z_k = \sigma\left(\sum_{j=1}^{4096} W^{(1,j,k)} x_j + b^{(1,k)}\right)$$
$$= \sigma\left(W^{(1,*,k)} \cdot x + b^{(1,k)}\right)$$



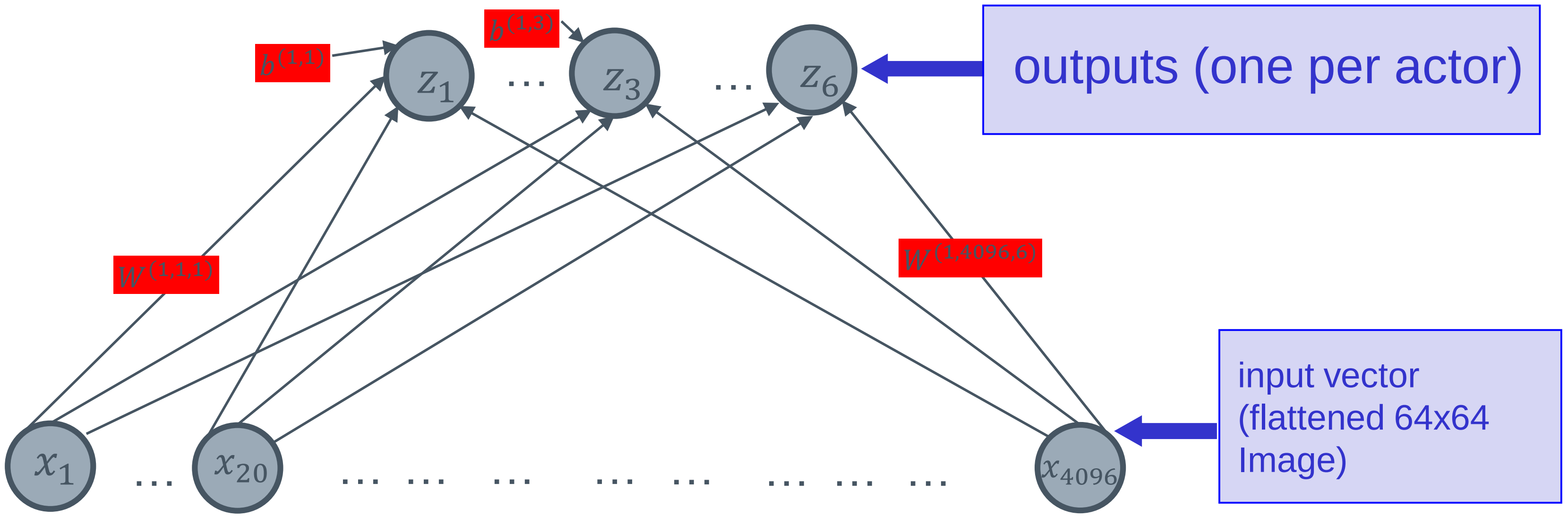
$$\boxed{h_\theta = h_{W,b}}$$



The transformation with $\sigma$ is not necessary here, but will be useful later

## Training a neural network

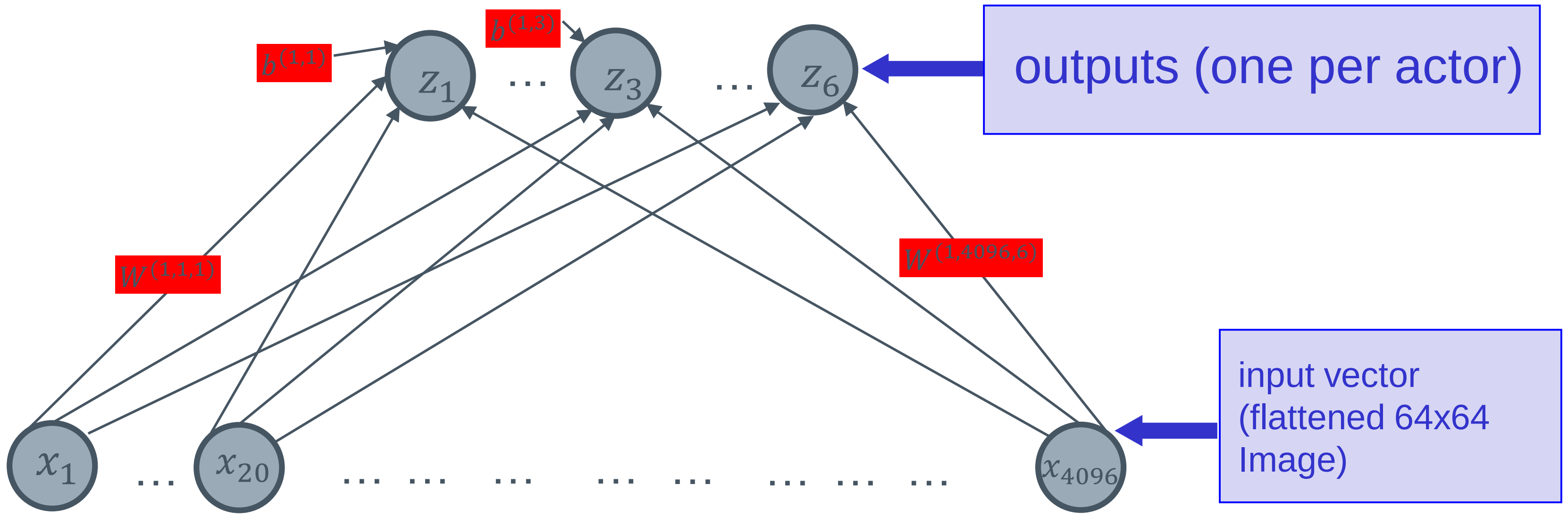
› Adjust the W's ($4096 \times 6$ coefs) and b's (6 coefs)
  – Try to make it so that if
    $x$ is an image of actor 1, $z$ is as close as possible to (1, 0, 0, 0, 0, 0)
    $x$ is an image of actor 2, $z$ is as close as possible to (0, 1, 0, 0, 0, 0)
    ……



# Face recognition

› Compute the z for a new image x

› If $z_k$ is the largest output, output name *k*



## An interpretation

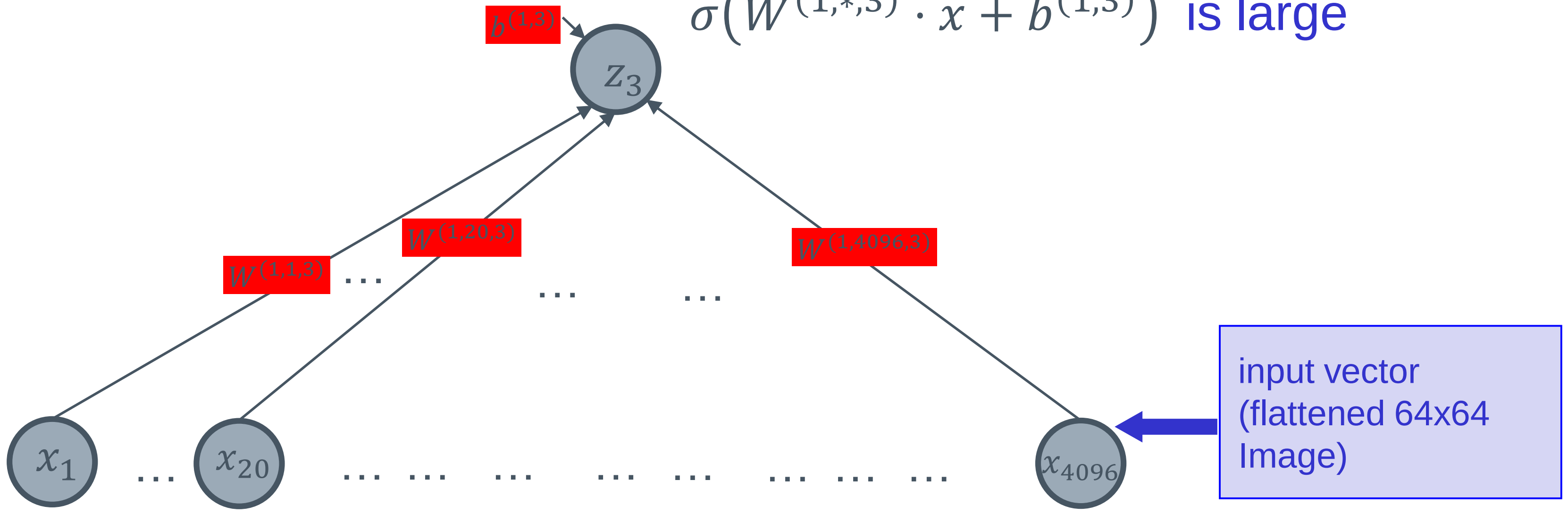
$z_1$ is large if $W^{(1,*,1)} \cdot x$ is large
$z_2$ is large if $W^{(1,*,2)} \cdot x$ is large

$z_3$ is large if $W^{(1,*,3)} \cdot x$ is large

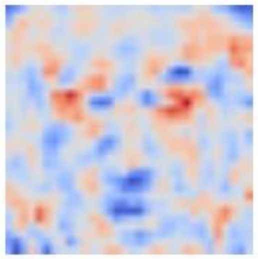
….

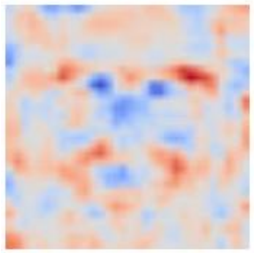
$W^{(1,*,1)}$, $W^{(1,*,2)}$, …, $W^{(1,*,6)}$ are *templates* for the faces of actor 1, actor 2, …, actor 6

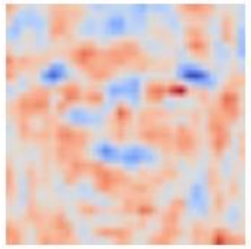
Actor 3 neuron activated:
$\sigma\left(W^{(1,*,3)} \cdot x + b^{(1,3)}\right)$ is large

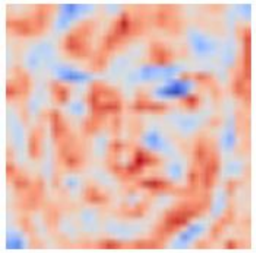


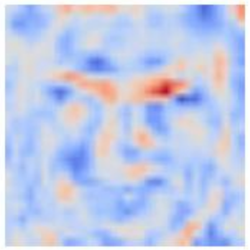
## Visualizing the parameters W

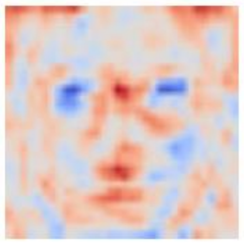

Baldwin
$W^{(1,*,1)}$

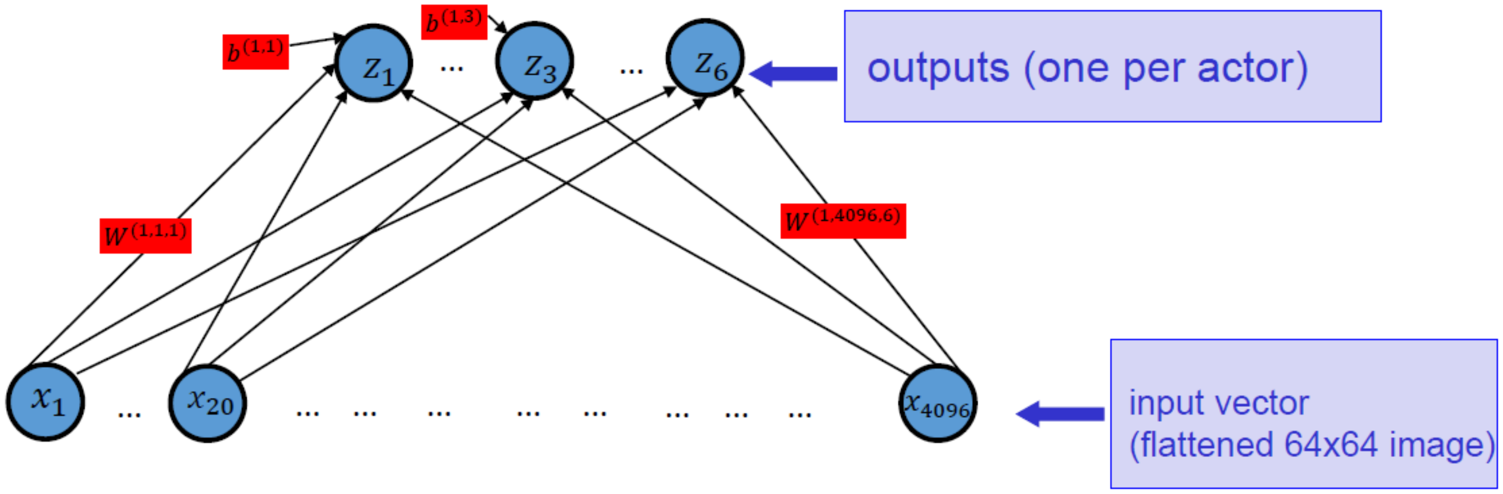

Carrel
$W^{(1,*,2)}$


Hader
$W^{(1,*,3)}$

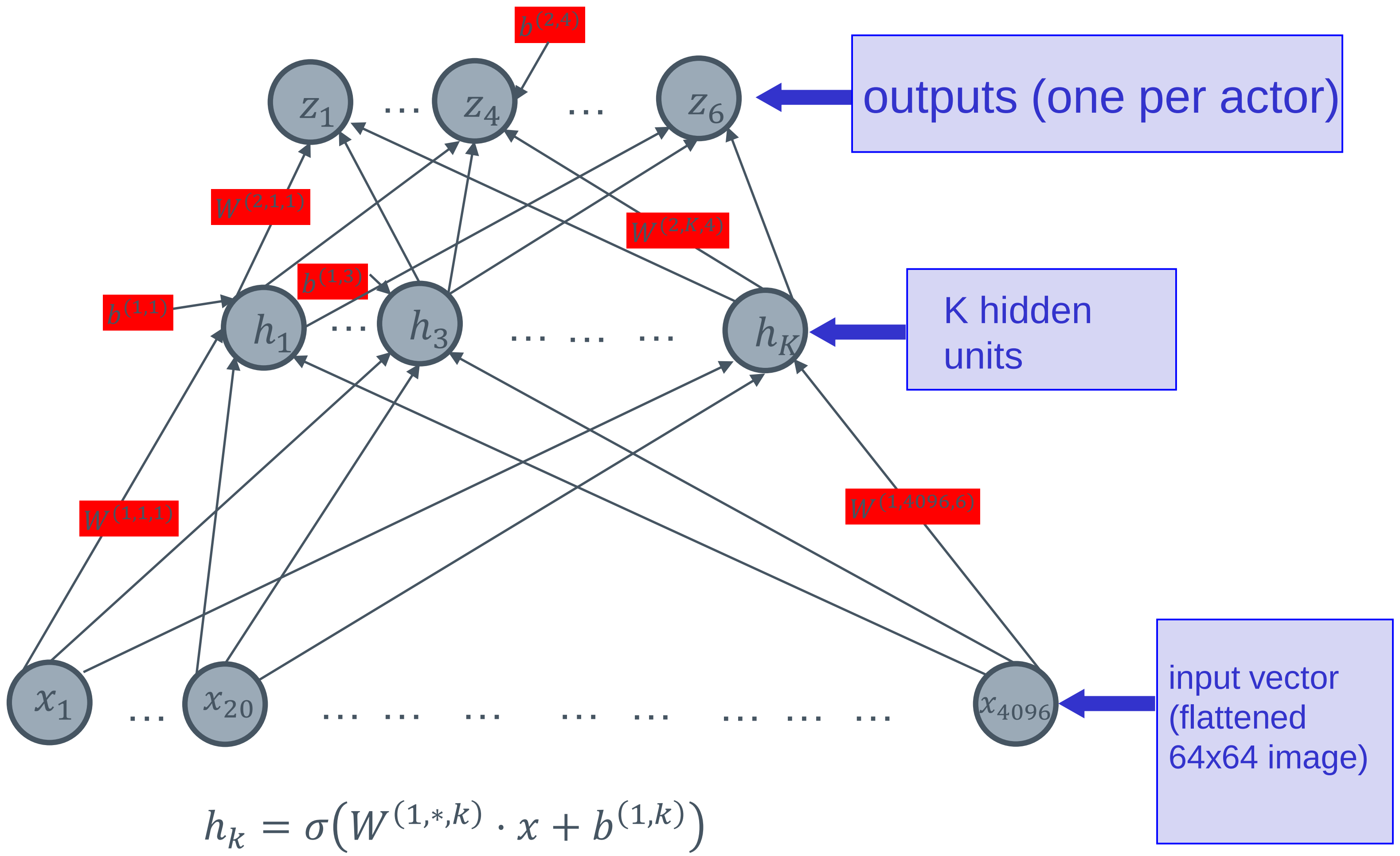

Ferrera
$W^{(1,*,4)}$


Drescher
$W^{(1,*,5)}$


Chenoweth
$W^{(1,*,6)}$



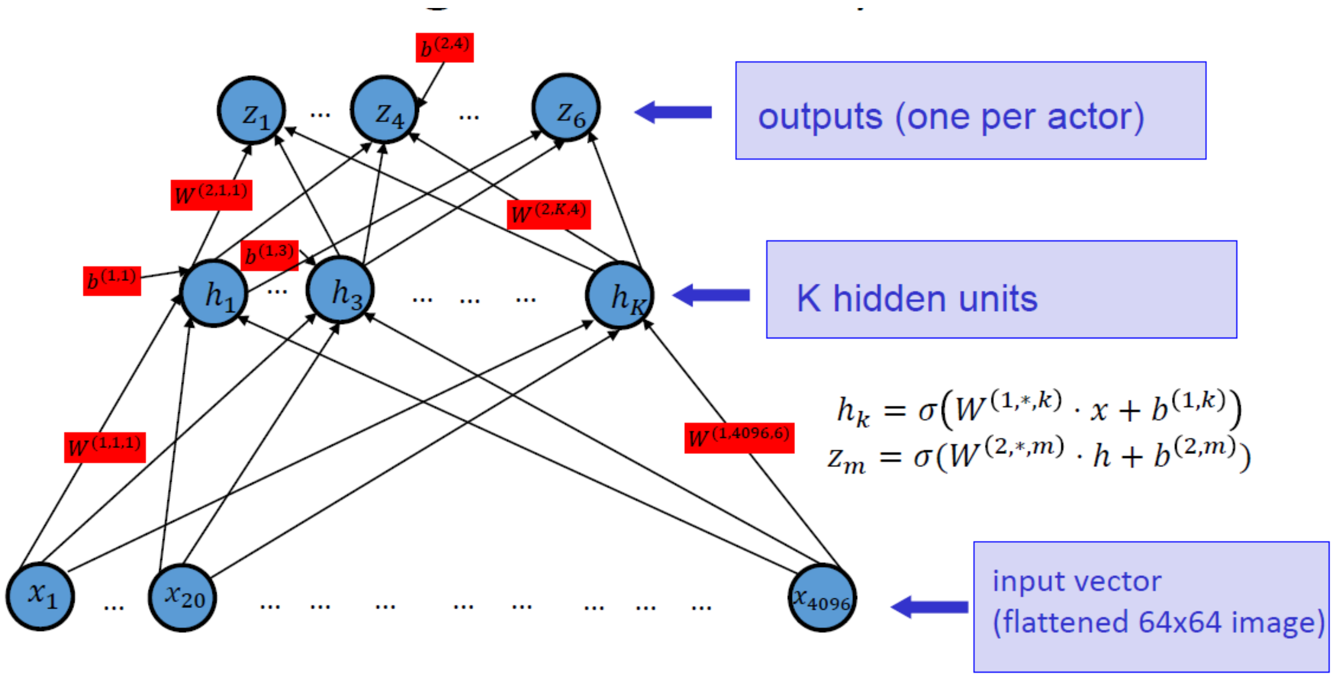
# Deep Neural Networks: Introducing Hidden Layers



$$h_k = \sigma\left(W^{(1,*,k)} \cdot x + b^{(1,k)}\right)$$
$$z_m = \sigma(W^{(2,*,m)} \cdot h + b^{(2,m)})$$

## Why a hidden layer?

- Instead of checking whether $x$ looks like one of 6 templates, we'll be checking whether $x$ looks like one of $K$ templates, for a large $K$
  - If template $k$ (i.e., $W^{(1,*,k)}$) looks like actor 6, $W^{(2,k,6)}$ will be large



$$h_k = \sigma\left(W^{(1,*,k)} \cdot x + b^{(1,k)}\right)$$
$$z_m = \sigma(W^{(2,*,m)} \cdot h + b^{(2,m)})$$

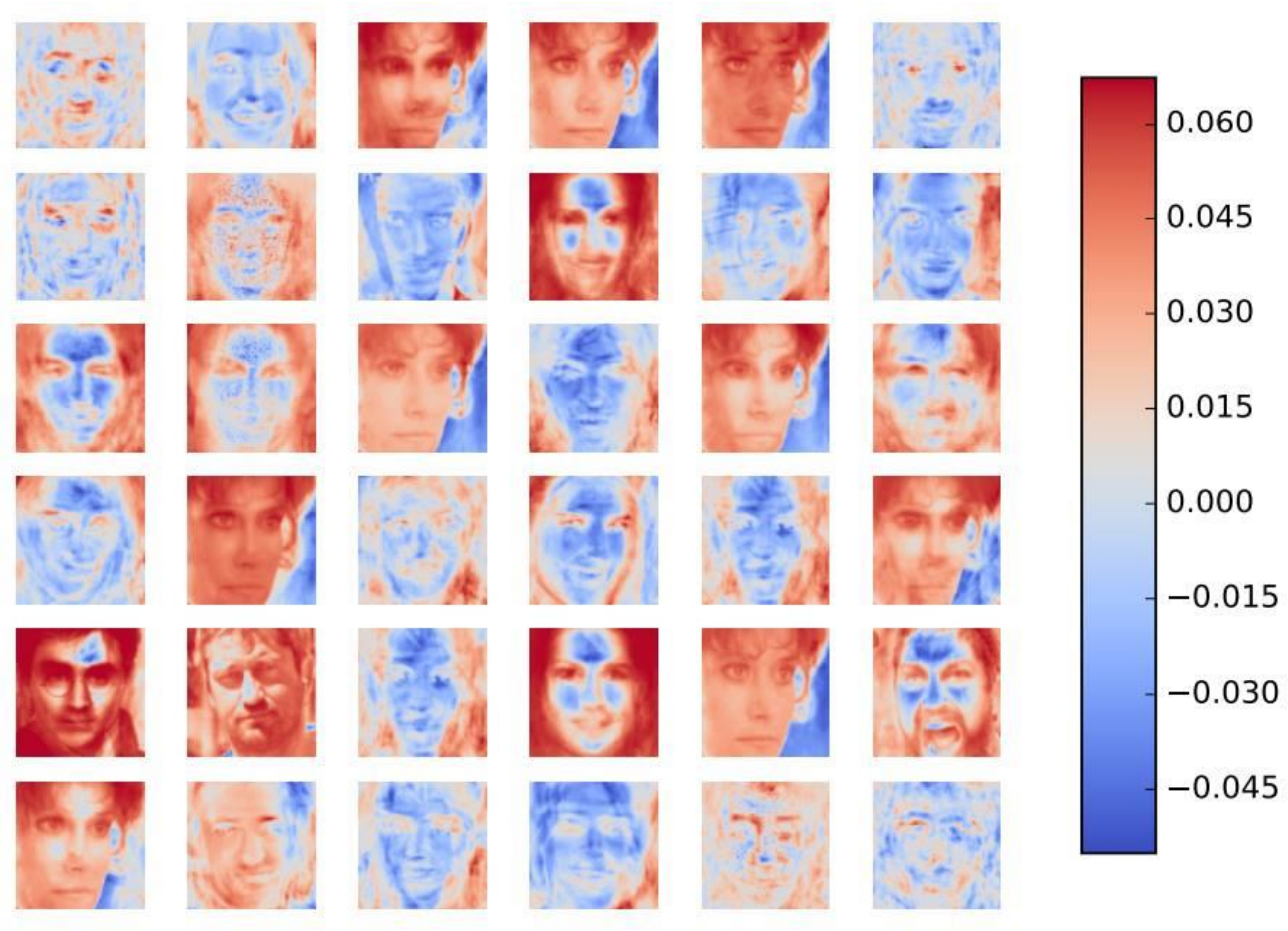
## Recap: Face Recognition with ML

- 1-Nearest-Neighbor: match $x$ to all the images in the training set
- 0-hidden-layer neural network*: match $x$ to several templates, with one template per actor
  - The templates work better than any individual photo
- 1-hidden-layer neural network: match $x$ to $K$ templates
  - The templates work better than any individual photo
  - More templates means better accuracy on the training set

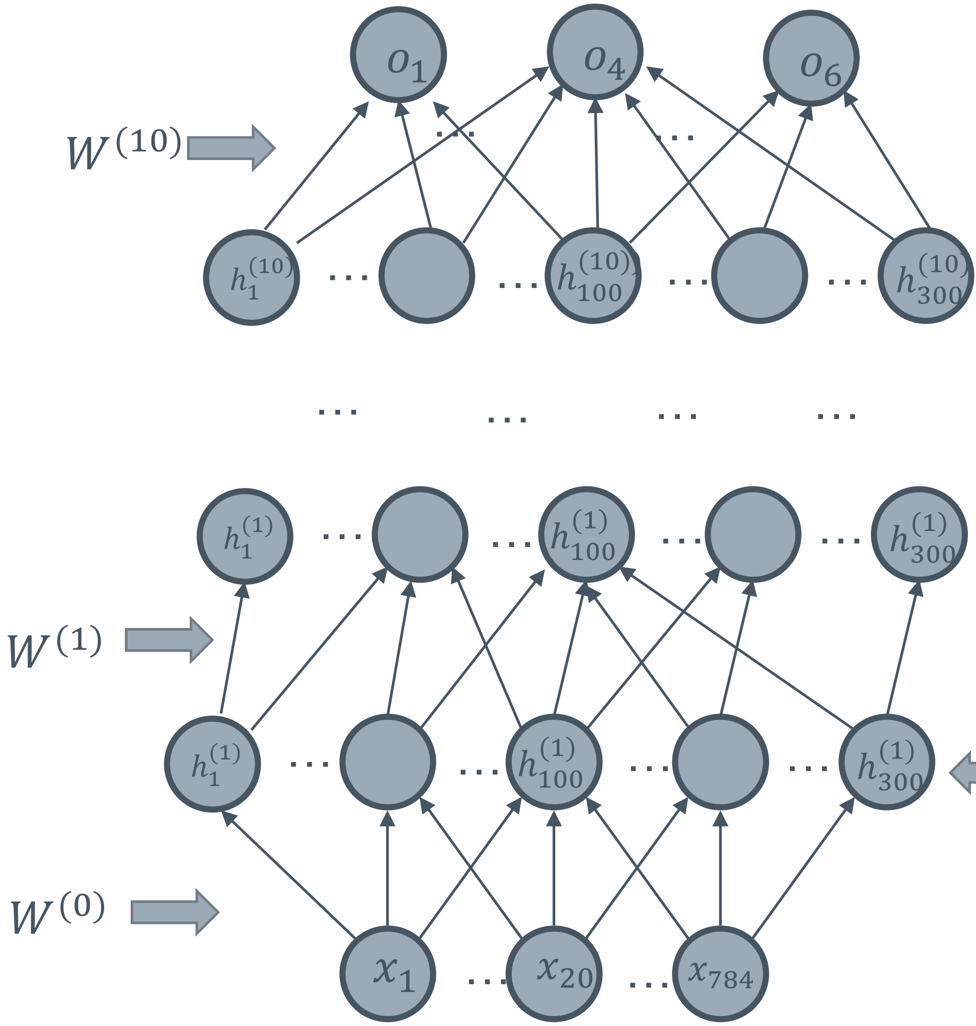
*A.K.A. multinomial logistic regression

** With minor modifications made to make this lecture clearer

## Visualizing a One-Hidden-Layer NN



# Deep Learning: More hidden layers!



Templates for *template matchers* (combinations of very simple shapes)

Templates for the input (very simple shapes)

## Deep Neural Networks as a Model of Computation

- Most people's first instinct when building a face classifier is to write a complicated computer program
- A deep neural network *is* a computer program:

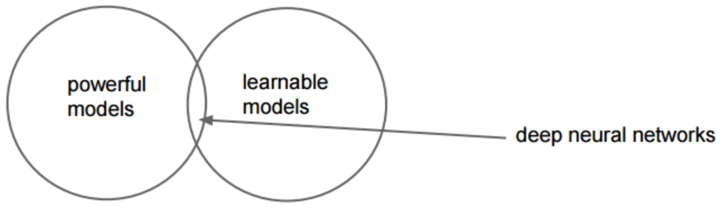
```
h1 = f1(x)
h2 = f2(h1)
h3 = f3(h2)
…
h9 = f9(h8)
```

- Can think of every layer of a neural network as one step of a parallel computation
- Features/templates are the functions that are applied to the previous layers
- Learning features ⇔ Learning what function to apply at step t of the algorithm

## Deep Neural Networks

› Can perform a wide range of computation

› Can be learned automatically
  – (using gradient descent)



- Powerful but not (computer) learnable: Python
  - Can't make a learning algorithm that takes lots of inputs and outputs and produces Python code that generates the outputs on new inputs
    - (But can do it with simpler languages!)
- Learnable but not powerful:
  - Logistic regression
  - Deep Neural Networks that aren't deep enough

Graphic and idea by Ilya Sutskever

## The *Deep Learning Hypothesis*

- Human perception is fast
  - (Human) neurons fire at most 100 times a second
  - Humans can solve simple perceptual tasks in 0.1 seconds
    - So out neurons fire in a sequence of 10 times at most

Anything a human can do in 0.1 seconds, a big 10-layer neural network can do, too!

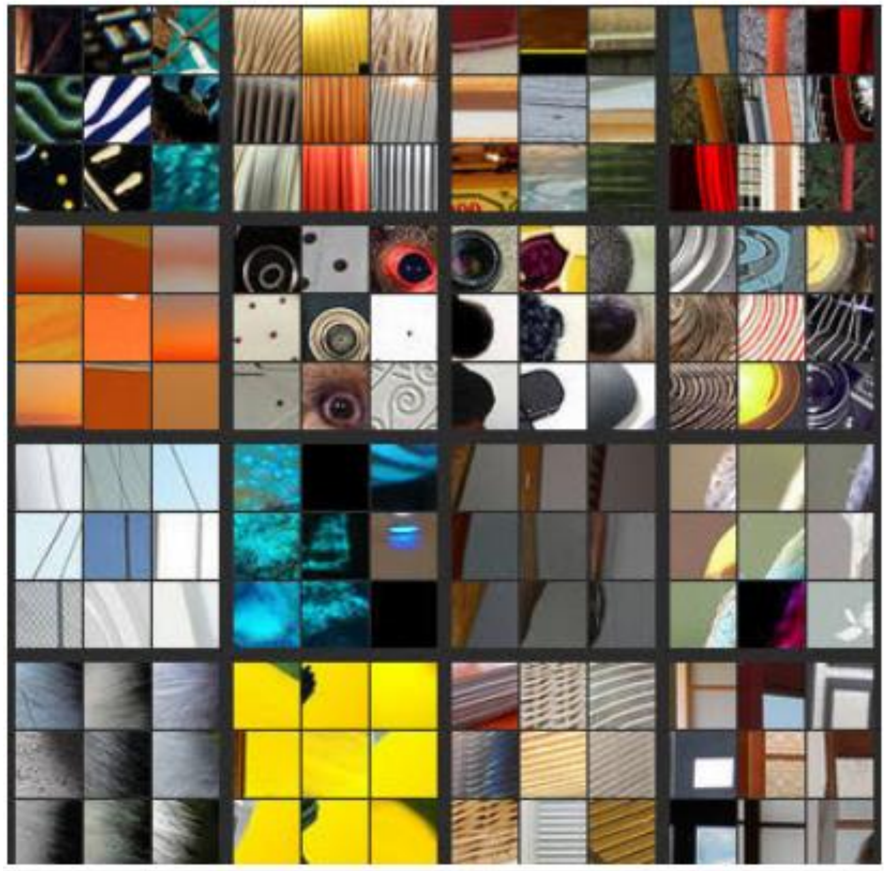
- Success stories:
  - Classifying images of objects
  - Classifying Go positions as good or bad

## What are the hidden units doing?

## What are the hidden units doing?

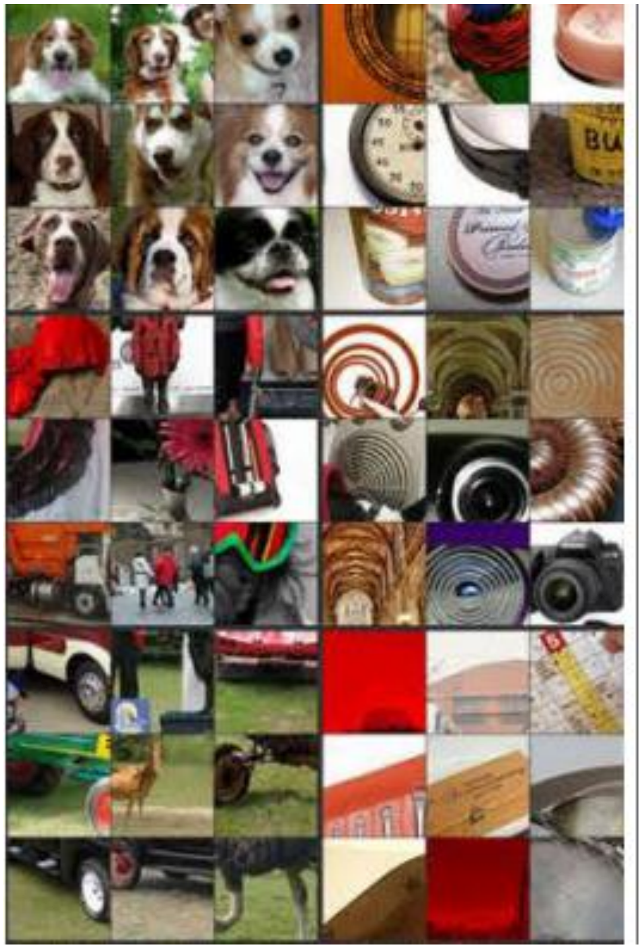
- Find the images in the dataset that activate the units the *most*
- *Let's see some visualizations of neurons of a large deep network trained to recognize objects in images*
  - The network classifies images as one of 1000 objects (sample objects: toy poodle, flute, forklift, goldfish…)
  - The network has 8 layers
  - Note: more tricks were used in designing the networks than we have time to mention. In particular, a *convolutional* architecture is crucial

## Units in Layer 3



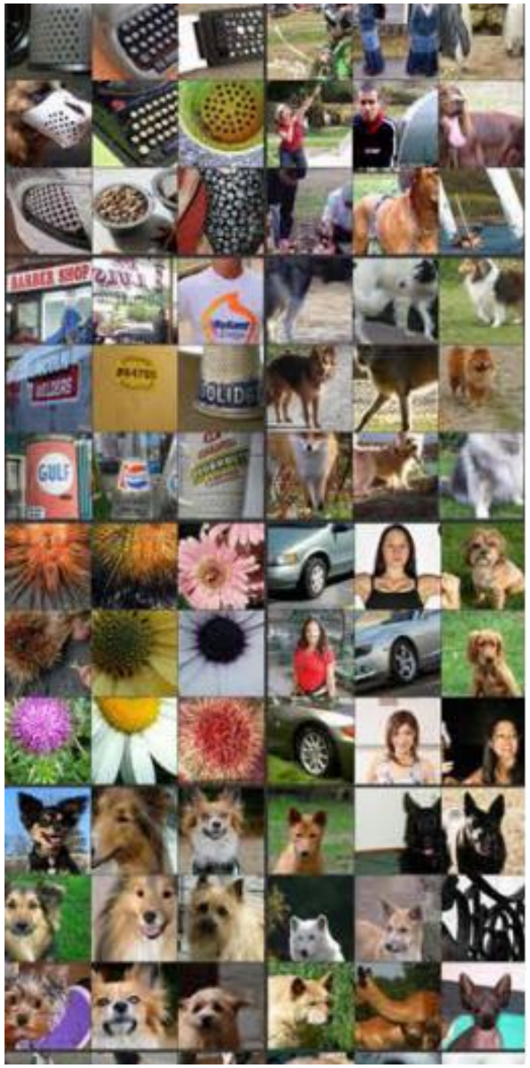
Matthew Zeiler and Rob Fergus, “Visualizing and Understanding Convolutional Networks” (ECCV 2014)

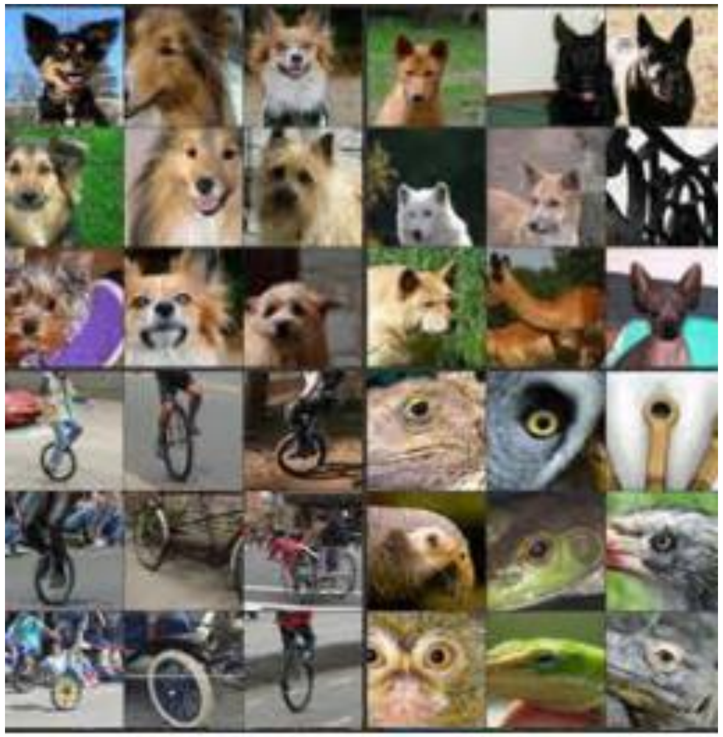
## Units in Layer 4



Matthew Zeiler and Rob Fergus, “Visualizing and Understanding Convolutional Networks” (ECCV 2014)

## Units in Layer 5





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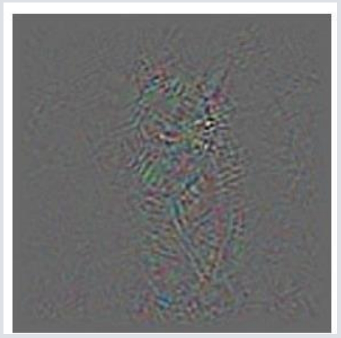
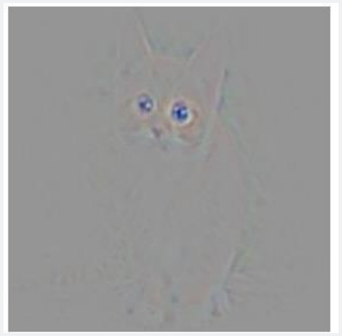
## Which pixels are responsible for the output?

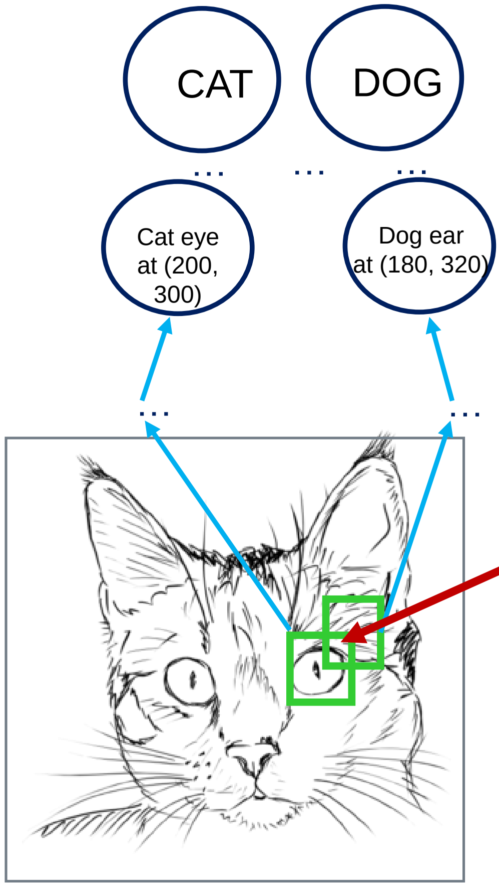
- For each pixel in a particular image ask:
  - If I changed the pixel $j$ by a little bit, how would that influence the output $i$?
  - Equivalent to asking: what's the gradient $\frac{\partial output_i}{\partial input_j}$
  - We can visualize why a particular output was chosen by the network by computing $\frac{\partial output_i}{\partial input_j}$ for every j, and displaying that as an image ("saliency map")

## Gradient and Guided Backpropagation

| | |
|---|---|
| Image I | |
| $\frac{\partial \text{Cat-Neuron}}{\partial I}$ |  |
| Guided Backpropagation visualization |  |

Graphic and idea by Andrej Karpathy

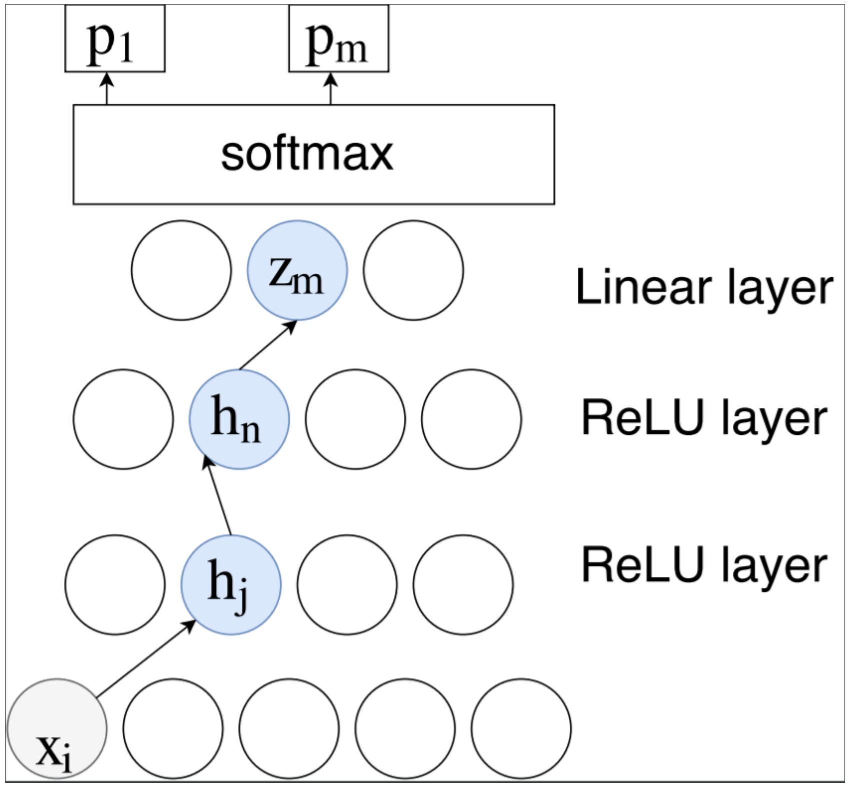
## Why the gradient with respect to the input is noisy



Pixel provides both positive (via a cat eye detection) and negative (via absence of cat eye detection) evidence for a cat in the image

## Guided backpropagation

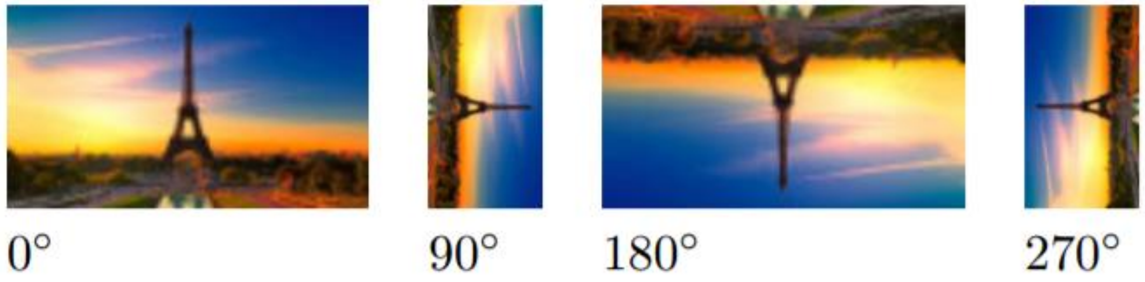
- Instead of computing $\frac{\partial p_m}{\partial x}$, only consider paths from $x$ to $p_m$ where the weights are positive and all the units are positive (and greater than 0). Compute this modified version of $\frac{\partial p_m}{\partial x}$
- Only consider evidence for neurons being active, discard evidence for neurons having to be not active



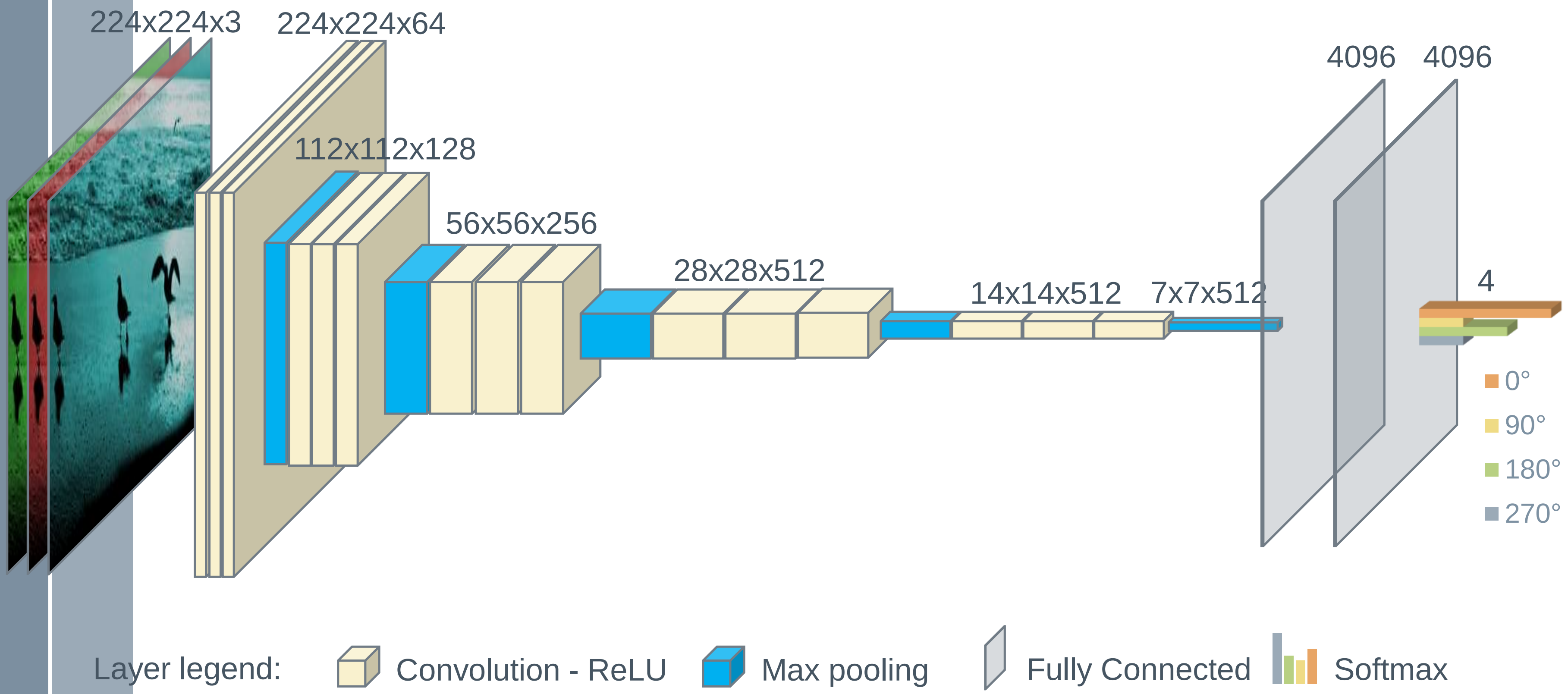
Questions?

## Application: Photo Orientation

- Detect the correct orientation of a consumer photograph
- Input photo is rotated by 0°, 90°, 180° or 270°
- Help speed up the digitization of analog photos
- Need correctly oriented photos as inputs for other systems

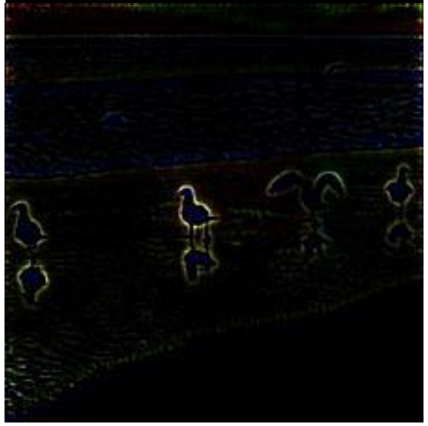
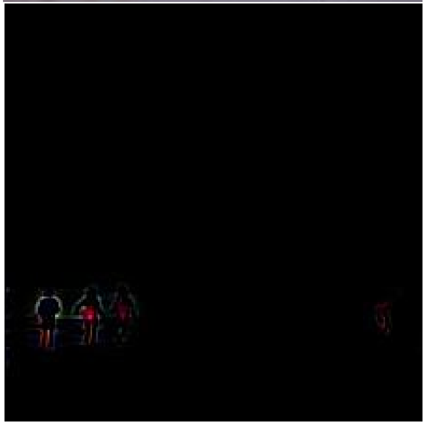


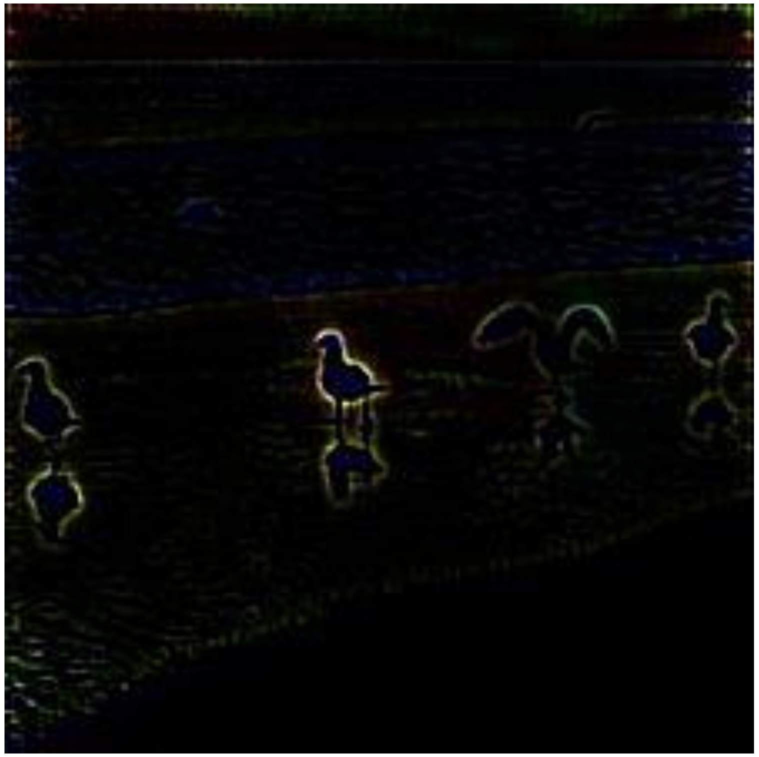
# A Neural Network for Photo Orientation

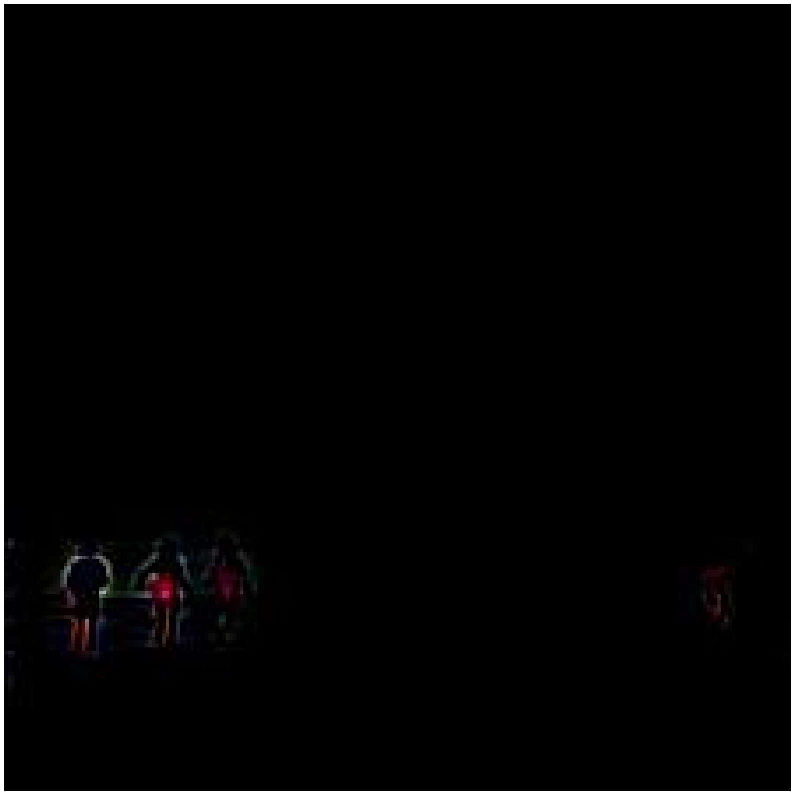


## Correctly Oriented Photos

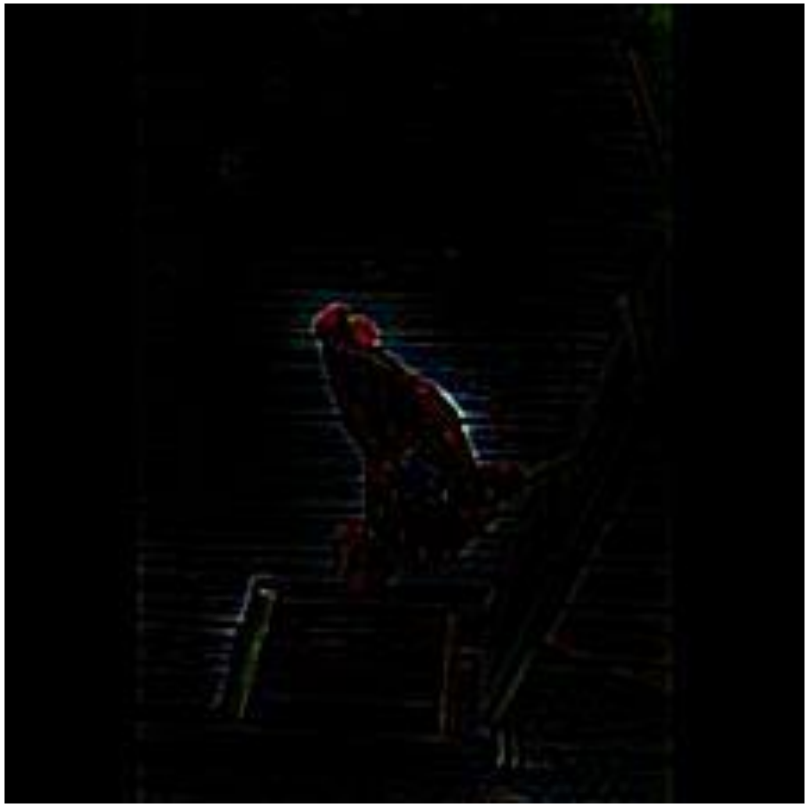
- Display pixels that provide direct positive evidence for 0°

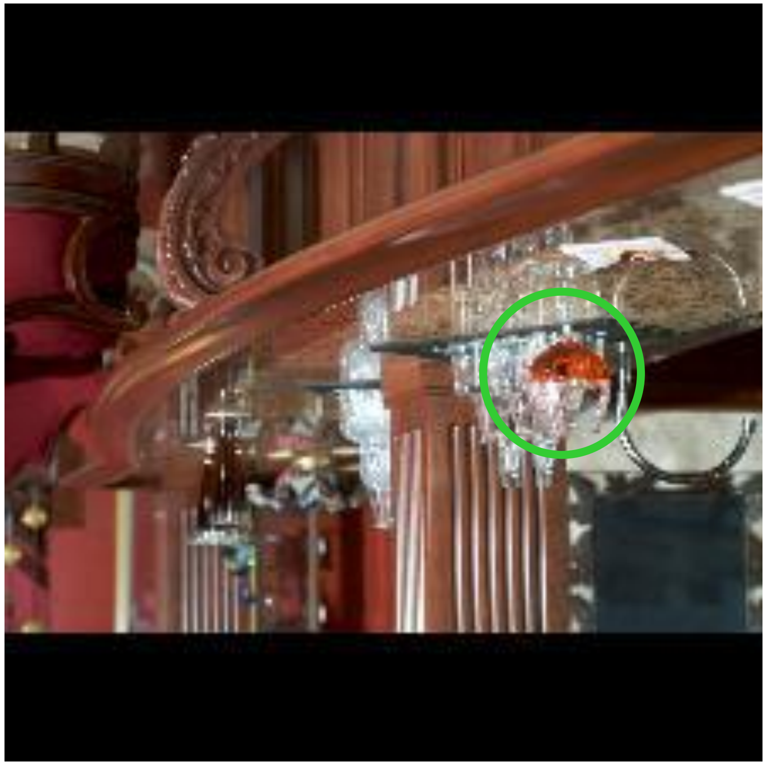







## Incorrectly-oriented photos





Questions?