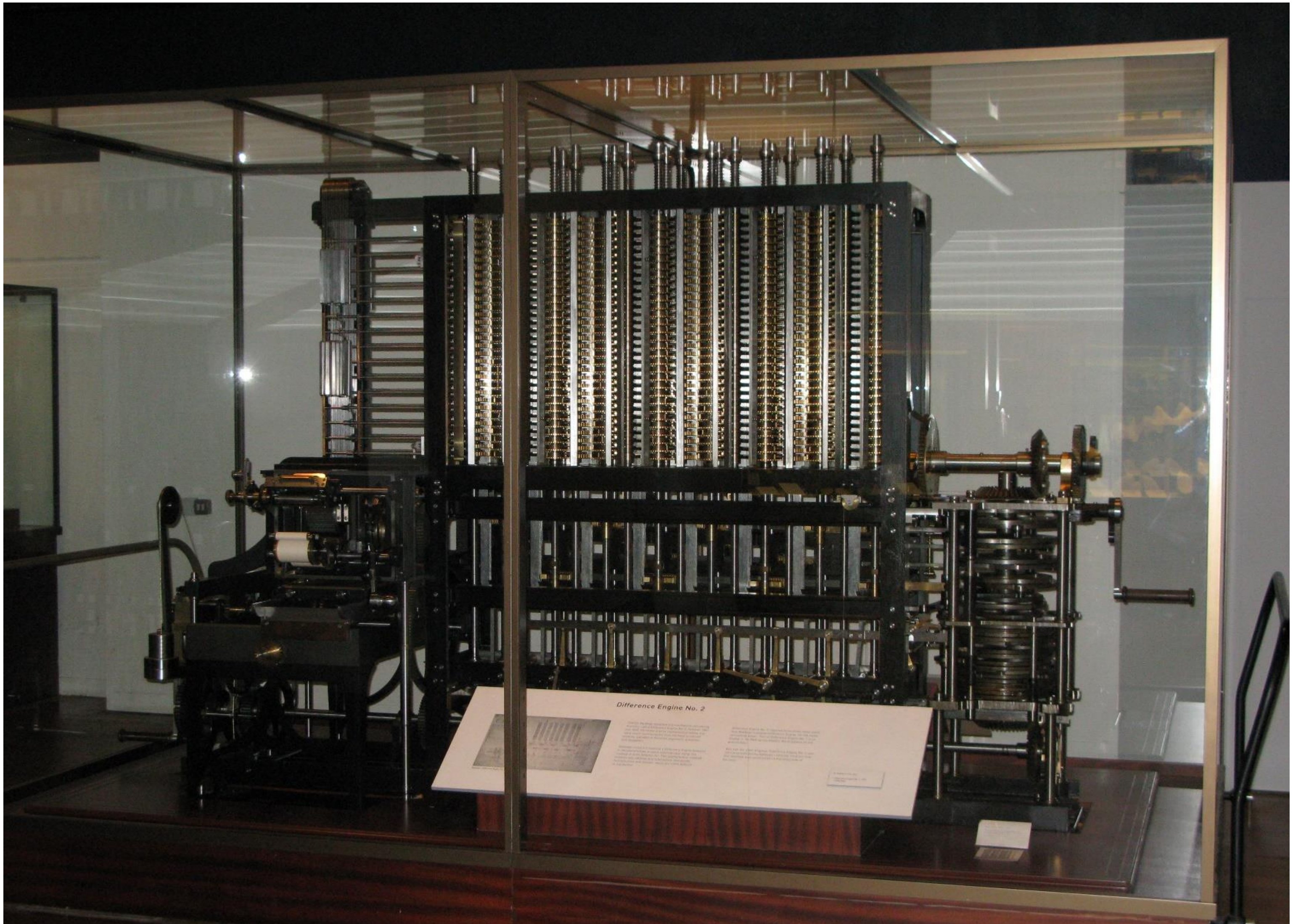
UNIVERSITY OF TORONTO

Ada Lovelace Day

Ada, Countess of Lovelace, 1840

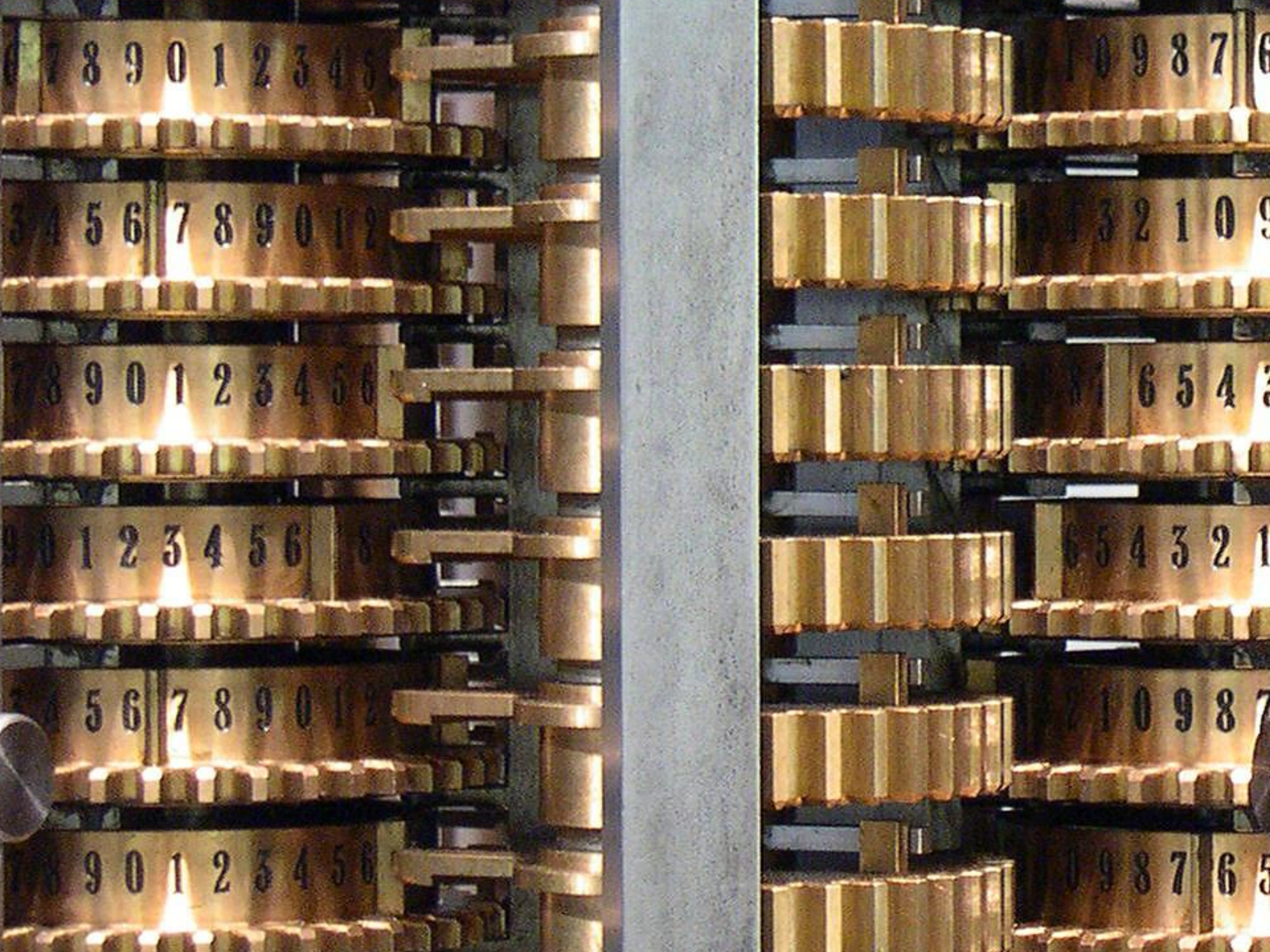
The Difference Engine



The Difference Engine

- A mechanical calculator that was able to automatically compute values of arbitrary polynomials
- Designed by Charles Babbage (the Lucasian Professor of Mathemetics at Cambridge*)

* At one time, Isaac Newton’s job



The Analytical Engine

- An extension of the ideas of the Difference Engine by Babbage
- Could do loops

The world's first computer program

- Ada Lovelace wrote about the Analytical Engine. To explain its utility, she wrote a complex program for it. The program computed Bernoulli Numbers

$$\left.\begin{array}{c} 0 = -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + \mathrm{B}_1 \left(\frac{2n}{2}\right) + \mathrm{B}_3 \left(\frac{2n\cdot(2n-1)\cdot(2n-2)}{2\cdot3\cdot4}\right) + \\ + \mathrm{B}_5 \left(\frac{2n\cdot(2n-1)\ldots(2n-4)}{2\cdot3\cdot4\cdot5\cdot6}\right) + \cdots + \mathrm{B}_{2n-1} \end{array}\right\}$$

$$\left.\begin{array}{c} 0 = -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + \mathrm{B}_1 \left(\frac{2n}{2}\right) + \mathrm{B}_3 \left(\frac{2n\cdot(2n-1)\cdot(2n-2)}{2\cdot3\cdot4}\right) + \\ +\mathrm{B}_5 \left(\frac{2n\cdot(2n-1)\ldots(2n-4)}{2\cdot3\cdot4\cdot5\cdot6}\right) + \cdots + \mathrm{B}_{2n-1} \end{array}\right\}$$

Idea: first compute $B_1 \ldots B_k$, and the use a loop to compute B_{k+1}

The World's First Computer Program

Number of Operation	Nature of Operation	Variables acted upon	Variables receiving results	Indication of change in the value on any Variable	Statement of Results	Data			Working Variables									
						$^1V_1$	$^1V_2$	$^1V_3$	$^0V_4$	$^0V_5$	$^0V_6$	$^0V_7$	$^0V_8$	$^0V_9$	$^0V_{10}$	$^0V_{11}$	$^0V_{12}$	$^0V_{13}\ldots$
						○	○	○	○	○	○	○	○	○	○	○	○	○
						0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0
						1	2	4	0	0	0	0	0	0	0	0	0	0
						[1]	[2]	[n]	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]	[ ]
1	×	$^1V_2 \times {}^1V_3$	$^1V_4$, $^1V_5$, $^1V_6$	$^1V_2 = {}^1V_2$ $^1V_3 = {}^1V_3$	$= 2n$	......	2	n	$2n$	$2n$	$2n$							
2	−	$^1V_4 - {}^1V_1$	$^2V_4$	$^1V_4 = {}^2V_4$ $^1V_1 = {}^1V_1$	$= 2n - 1$	1	......	......	$2n - 1$									
3	+	$^1V_5 + {}^1V_1$	$^2V_5$	$^1V_5 = {}^2V_5$ $^1V_1 = {}^1V_1$	$= 2n + 1$	1	......	......	......	$2n + 1$								
4	÷	$^2V_5 \div {}^2V_4$	$^1V_{11}$	$^2V_5 = {}^0V_5$ $^2V_4 = {}^0V_4$	$= \frac{2n-1}{2n+1}$	......	......	......	0	0	......	......	......	......	......	$\frac{2n-1}{2n+1}$		
5	÷	$^1V_{11} \div {}^1V_2$	$^2V_{11}$	$^1V_{11} = {}^2V_{11}$ $^1V_2 = {}^1V_2$	$= \frac{1}{2} \cdot \frac{2n-1}{2n+1}$	......	2	......	......	......	......	......	......	......	......	$\frac{1}{2} \cdot \frac{2n-1}{2n+1}$		
6	−	$^0V_{13} - {}^2V_{11}$	$^1V_{13}$	$^2V_{11} = {}^0V_{11}$ $^0V_{13} = {}^1V_{13}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$	......	......	......	......	......	......	......	......	......	......	0	......	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$
7	−	$^1V_3 - {}^1V_1$	$^1V_{10}$	$^1V_3 = {}^1V_3$ $^1V_1 = {}^1V_1$	$= n - 1 (= 3)$	1	......	n	......	......	......	......	......	......	$n - 1$			
8	+	$^1V_2 + {}^0V_7$	$^1V_7$	$^1V_2 = {}^1V_2$ $^0V_7 = {}^1V_7$	$= 2 + 0 = 2$	......	2	......	......	......	......	2						
9	÷	$^1V_6 \div {}^1V_7$	$^3V_{11}$	$^1V_6 = {}^1V_6$ $^0V_{11} = {}^3V_{11}$	$= \frac{2n}{2} = A_1$	......	......	......	......	......	$2n$	2	......	......	......	$\frac{2n}{2} = A_1$		
10	×	$^1V_{21} \times {}^3V_{11}$	$^1V_{12}$	$^1V_{21} = {}^1V_{21}$ $^3V_{11} = {}^3V_{11}$	$= B_1 \cdot \frac{2n}{2} = B_1A_1$	......	......	......	......	......	......	......	......	......	......	$\frac{2n}{2} = A_1$	$B_1 \cdot \frac{2n}{2} = B_1A_1$	......
11	+	$^1V_{12} + {}^1V_{13}$	$^2V_{13}$	$^1V_{12} = {}^0V_{12}$ $^1V_{13} = {}^2V_{13}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$	......	......	......	......	......	......	......	......	......	......	......	0	$\left\{-\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}\right.$
12	−	$^1V_{10} - {}^1V_1$	$^2V_{10}$	$^1V_{10} = {}^2V_{10}$ $^1V_1 = {}^1V_1$	$= n - 2 (= 2)$	1	......	......	......	......	......	......	......	......	$n - 2$			
13	−	$^1V_6 - {}^1V_1$	$^2V_6$	$^1V_6 = {}^2V_6$ $^1V_1 = {}^1V_1$	$= 2n - 1$	1	......	......	......	......	$2n - 1$							
14	+	$^1V_1 + {}^1V_7$	$^2V_7$	$^1V_1 = {}^1V_1$ $^1V_7 = {}^2V_7$	$= 2 + 1 = 3$	1	......	......	......	......	......	3						
15	÷	$^2V_6 \div {}^2V_7$	$^1V_8$	$^2V_6 = {}^2V_6$ $^2V_7 = {}^2V_7$	$= \frac{2n-1}{3}$	......	......	......	......	......	$2n - 1$	3	$\frac{2n-1}{3}$					
16	×	$^1V_8 \times {}^3V_{11}$	$^4V_{11}$	$^1V_8 = {}^0V_8$ $^3V_{11} = {}^4V_{11}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3}$	......	......	......	......	......	......	......	0	......	......	$\frac{2n}{2} \cdot \frac{2n-1}{3}$		
17	−	$^2V_6 - {}^1V_1$	$^3V_6$	$^2V_6 = {}^3V_6$ $^1V_1 = {}^1V_1$	$= 2n - 2$	1	......	......	......	......	$2n - 2$							
18	+	$^1V_1 + {}^2V_7$	$^3V_7$	$^2V_7 = {}^3V_7$ $^1V_1 = {}^1V_1$	$= 3 + 1 = 4$	1	......	......	......	......	......	4						
19	÷	$^3V_6 \div {}^3V_7$	$^1V_9$	$^3V_6 = {}^3V_6$ $^3V_7 = {}^3V_7$	$= \frac{2n-2}{4}$	......	......	......	......	......	$2n - 2$	4	......	$\frac{2n-2}{4}$	......			
20	×	$^1V_9 \times {}^4V_{11}$	$^5V_{11}$	$^1V_9 = {}^0V_9$ $^4V_{11} = {}^5V_{11}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = A_3$	......	......	......	......	......	......	......	......	0		$\left\{\frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4}\right\} = A_3$		
21	×	$^1V_{22} \times {}^5V_{11}$	$^0V_{12}$	$^1V_{22} = {}^1V_{22}$	$= B_3 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = B_3A_3$	......	......	......	......	......	......	......	......	......	......	0	B_3A_3	......