1 Forward Step: Example 1

The matrix is currently:
[[ 0. 0. 1. 0. 2.]
 [ 1. 0. 2. 3. 4.]
 [ 3. 0. 4. 2. 1.]
 [ 1. 0. 1. 1. 2.]]

Now looking at row 0
Swapping rows 0 and 1 so that entry 0 in the current row is non-zero
The matrix is currently:
[[ 1. 0. 2. 3. 4.]
 [ 0. 0. 1. 0. 2.]
 [ 3. 0. 4. 2. 1.]
 [ 1. 0. 1. 1. 2.]]

Adding row 0 to rows below it to eliminate coefficients in column 0
The matrix is currently:
[[ 1. 0. 2. 3. 4.]
 [ 0. 0. 1. 0. 2.]
 [ 0. 0. -2. -7. -11.]
 [ 0. 0. -1. -2. -2.]]

Now looking at row 1
Swapping rows 1 and 1 so that entry 2 in the current row is non-zero
The matrix is currently:
[[ 1. 0. 2. 3. 4.]
 [ 0. 0. 1. 0. 2.]
 [ 0. 0. -2. -7. -11.]
 [ 0. 0. -1. -2. -2.]]

Adding row 1 to rows below it to eliminate coefficients in column 2
The matrix is currently:
[[ 1. 0. 2. 3. 4.]
 [ 0. 0. 1. 0. 2.]
 [ 0. 0. 0. -7. -7.]
 [ 0. 0. 0. -2. 0.]]

Now looking at row 2
Swapping rows 2 and 2 so that entry 3 in the current row is non-zero
The matrix is currently:
[[ 1. 0. 2. 3. 4.]
 [ 0. 0. 1. 0. 2.]
 [ 0. 0. 0. -7. -7.]
 [ 0. 0. 0. -2. 0.]]

Adding row 2 to rows below it to eliminate coefficients in column 3
The matrix is currently:
[[ 1. 0. 2. 3. 4.]
 [ 0. 0. 1. 0. 2.]
 [ 0. 0. 0. -7. -7.]
 [ 0. 0. 0. 0. 2.]]
Now looking at row 3

Swapping rows 3 and 3 so that entry 4 in the current row is non-zero

The matrix is currently:

\[
\begin{bmatrix}
1 & 0 & 2 & 3 & 4 \\
0 & 0 & 1 & 0 & 2 \\
0 & 0 & 0 & -7 & -7 \\
0 & 0 & 0 & 0 & 2 \\
\end{bmatrix}
\]

Adding row 3 to rows below it to eliminate coefficients in column 4

The matrix is currently:

\[
\begin{bmatrix}
1 & 0 & 2 & 3 & 4 \\
0 & 0 & 1 & 0 & 2 \\
0 & 0 & 0 & -7 & -7 \\
0 & 0 & 0 & 0 & 2 \\
\end{bmatrix}
\]

Done with the forward step
2 Forward Step and Backward Step: Example 2

The matrix is currently:
[[ 1  -2  3  22]
 [ 3  10  1  314]
 [ 1  5  3  92]]
================================================================================
Now performing the forward step
Now looking at row 0
Swapping rows 0 and 0 so that entry 0 in the current row is non-zero
The matrix is currently:
[[ 1  -2  3  22]
 [ 3  10  1  314]
 [ 1  5  3  92]]
================================================================================
Adding row 0 to rows below it to eliminate coefficients in column 0
The matrix is currently:
[[ 1.  -2.  3.  22.]
 [ 0.  16.  -8.  248.]
 [ 0.  7.  0.  70.]]
================================================================================
Now looking at row 1
Swapping rows 1 and 1 so that entry 1 in the current row is non-zero
The matrix is currently:
[[ 1.  -2.  3.  22.]
 [ 0.  16.  -8.  248.]
 [ 0.  7.  0.  70.]]
================================================================================
Adding row 1 to rows below it to eliminate coefficients in column 1
The matrix is currently:
[[ 1.  -2.  3.  22.]
 [ 0.  16.  -8.  248.]
 [ 0.  0.  3.5 -38.5]]
================================================================================
Now looking at row 2
Swapping rows 2 and 2 so that entry 2 in the current row is non-zero
The matrix is currently:
[[ 1.  -2.  3.  22.]
 [ 0.  16.  -8.  248.]
 [ 0.  0.  3.5 -38.5]]
================================================================================
Adding row 2 to rows below it to eliminate coefficients in column 2
The matrix is currently:
[[ 1.  -2.  3.  22.]
 [ 0.  16.  -8.  248.]
 [ 0.  0.  3.5 -38.5]]
================================================================================
The matrix is currently:
\[
\begin{bmatrix}
1 & -2 & 3 & 22 \\
0 & 16 & -8 & 248 \\
0 & 0 & 3.5 & -38.5
\end{bmatrix}
\]

Now performing the backward step
Adding row 2 to rows above it to eliminate coefficients in column 2
The matrix is currently:
\[
\begin{bmatrix}
1 & -2 & 0 & 55 \\
0 & 16 & 0 & 160 \\
0 & 0 & 3.5 & -38.5
\end{bmatrix}
\]

Adding row 1 to rows above it to eliminate coefficients in column 1
The matrix is currently:
\[
\begin{bmatrix}
1 & 0 & 0 & 75 \\
0 & 16 & 0 & 160 \\
0 & 0 & 3.5 & -38.5
\end{bmatrix}
\]

Adding row 0 to rows above it to eliminate coefficients in column 0
The matrix is currently:
\[
\begin{bmatrix}
1 & 0 & 0 & 75 \\
0 & 16 & 0 & 160 \\
0 & 0 & 3.5 & -38.5
\end{bmatrix}
\]

Now dividing each row by the leading coefficient
The matrix is currently:
\[
\begin{bmatrix}
1 & 0 & 0 & 75 \\
0 & 1 & 0 & 10 \\
0 & 0 & 1 & -11
\end{bmatrix}
\]