1 Basic laws of manipulating formal statements

identity laws	$P \wedge (Q \vee \neg Q)$	$\iff$	P
	$P \vee (Q \wedge \neg Q)$	$\iff$	P
idempotency laws	$P \wedge P$	$\iff$	P
	$P \vee P$	$\iff$	P
commutative laws	$P \wedge Q$	$\iff$	$Q \wedge P$
	$P \vee Q$	$\iff$	$Q \vee P$
	$(P \Leftrightarrow Q)$	$\iff$	$(Q \Leftrightarrow P)$
associative laws	$(P \wedge Q) \wedge R$	$\iff$	$P \wedge (Q \wedge R)$
	$(P \vee Q) \vee R$	$\iff$	$P \vee (Q \vee R)$
distributive laws	$P \wedge (Q \vee R)$	$\iff$	$(P \wedge Q) \vee (P \wedge R)$
	$P \vee (Q \wedge R)$	$\iff$	$(P \vee Q) \wedge (P \vee R)$
contrapositive	$P \Rightarrow Q$	$\iff$	$\neg Q \Rightarrow \neg P$
implication	$P \Rightarrow Q$	$\iff$	$\neg P \vee Q$
equivalence	$(P \Leftrightarrow Q)$	$\iff$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
double negation	$\neg(\neg P)$	$\iff$	P
DeMorgan's laws	$\neg(P \wedge Q)$	$\iff$	$\neg P \vee \neg Q$
	$\neg(P \vee Q)$	$\iff$	$\neg P \wedge \neg Q$
implication negation	$\neg(P \Rightarrow Q)$	$\iff$	$P \wedge \neg Q$
equivalence negation	$\neg(P \Leftrightarrow Q)$	$\iff$	$\neg(P \Rightarrow Q) \vee \neg(Q \Rightarrow P)$
quantifier negation	$\neg(\forall x \in D, P(x))$	$\iff$	$\exists x \in D, \neg P(x)$
	$\neg(\exists x \in D, P(x))$	$\iff$	$\forall x \in D, \neg P(x)$
quantifier distributive laws	$\forall x \in D, P(x) \wedge Q(x)$	$\iff$	$(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$
(where R does not contain variable x)	$\exists x \in D, P(x) \vee Q(x)$	$\iff$	$(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$
	$\forall x \in D, R \wedge Q(x)$	$\iff$	$R \wedge (\forall x \in D, Q(x))$
	$\forall x \in D, R \vee Q(x)$	$\iff$	$R \vee (\forall x \in D, Q(x))$
	$\exists x \in D, R \vee Q(x)$	$\iff$	$R \vee (\exists x \in D, Q(x))$
	$\exists x \in D, R \wedge Q(x)$	$\iff$	$R \wedge (\exists x \in D, Q(x))$
variable renaming	$\forall x \in D, P(x)$	$\iff$	$\forall y \in D, P(y)$
(where y does not appear in $P(x)$)	$\exists x \in D, P(x)$	$\iff$	$\exists y \in D, P(y)$

2 Rules of inference

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Introduction rules

[¬**I**] negation introduction

$$\begin{array}{l} \text{Assume } A \\ \quad\vdots \\ \quad\text{contradiction} \\ \hline \neg A \end{array}$$

[∧**I**] conjunction introduction

$$\begin{array}{l} A \\ B \\ \hline A \land B \end{array}$$

[∨**I**] disjunction introduction

$$\begin{array}{l} A \\ \hline A \lor B \\ B \lor A \end{array} \qquad \begin{array}{l} \\ \hline A \lor \neg A \end{array}$$

[⇒**I**] implication introduction

(direct)

$$\begin{array}{l} \text{Assume } A \\ \quad\vdots \\ \quad B \\ \hline A \Rightarrow B \end{array}$$

(indirect)

$$\begin{array}{l} \text{Assume } \neg B \\ \quad\vdots \\ \quad \neg A \\ \hline A \Rightarrow B \end{array}$$

[⇔**I**] equivalence/bi-implication introduction

$$\begin{array}{l} A \Rightarrow B \\ B \Rightarrow A \\ \hline A \Leftrightarrow B \end{array}$$

[∀**I**] universal introduction

$$\begin{array}{l} \text{Assume } a \in D \\ \quad\vdots \\ \quad P(a) \\ \hline \forall x \in D, P(x) \end{array}$$

[∃**I**] existential introduction

$$\begin{array}{l} P(a) \\ a \in D \\ \hline \exists x \in D, P(x) \end{array}$$

Elimination rules

[¬**E**] negation elimination

$$\begin{array}{l} \neg\neg A \\ \hline A \end{array} \qquad \begin{array}{l} A \\ \neg A \\ \hline \text{contradiction} \end{array}$$

[∧**E**] conjunction elimination

$$\begin{array}{l} A \land B \\ \hline A \\ B \end{array}$$

[∨**E**] disjunction elimination

$$\begin{array}{l} A \lor B \\ \neg A \\ \hline B \end{array} \qquad \begin{array}{l} A \lor B \\ \neg B \\ \hline A \end{array}$$

[⇒**E**] implication elimination

(Modus Ponens)

$$\begin{array}{l} A \Rightarrow B \\ A \\ \hline B \end{array}$$

(Modus Tollens)

$$\begin{array}{l} A \Rightarrow B \\ \neg B \\ \hline \neg A \end{array}$$

[⇔**E**] equivalence/bi-implication elimination

$$\begin{array}{l} A \Leftrightarrow B \\ \hline A \Rightarrow B \\ B \Rightarrow A \end{array}$$

[∀**E**] universal elimination

$$\begin{array}{l} \forall x \in D, P(x) \\ a \in D \\ \hline P(a) \end{array}$$

[∃**E**] existential elimination

$$\begin{array}{l} \exists x \in D, P(x) \\ \hline \text{Let } a \in D \text{ such that } P(a) \\ \quad\vdots \end{array}$$