(1) For each language $L_i$ below, give a Regular Expression $R_i$ and a Finite State Automaton $A_i$ such that $\mathcal{L}(R_i) = \mathcal{L}(A_i) = L_i$. For each RE and FSA, give a brief explanation of why they are correct (proofs not required). In all cases, the alphabet is $\Sigma = \{a, b\}$.

(a) $L_1 = \{s \in \Sigma^* : s$ contains $aba\}$
   
   e.g. $aba, aaababbb, abaaba \in L_1$

(b) $L_1 = \{s \in \Sigma^* : s$ doesn’t contain $aba\}$
   
   e.g. $aaabbb, bbabbb, abba \in L_2$

(c) $L_1 = \{a^n b^m : n, m \in \mathbb{N}$ and $n + m$ is even\}$
   
   e.g. $aabbb, \epsilon, aaab \in L_3$

(2) Prove that all finite languages can be described by a regular expression. You need to prove that for any finite language $L$, there is a regular expression $R$ such that $\mathcal{L}(R) = L$. If your alphabet is $\Sigma$ and the size of your language is $n$, we can say $L = \{s_1, s_2, \ldots, s_n\}$ where each $s_i \in \Sigma^*$. 