(1) Consider the following algorithm.

\#Precondition: \( n \) is a natural number
\#Postcondition: \( r = 3^n + 1 \)

\[
\begin{align*}
r & = 2 \\
i & = n \\
\text{while } i > 0 \\
r & = 3 \times r - 2 \\
i & = i - 1
\end{align*}
\]

(a) Prove that the following is a loop invariant:

\[
r = 3^{n-i} + 1
\]

(b) Prove partial correctness, i.e., prove:

\[
\text{Precondition } \land \text{ Termination } \rightarrow \text{Postcondition}
\]

(2) Consider the following algorithm for \texttt{mod} which returns the remainder of the division of \( n \) by \( d \).

\[
def \texttt{mod}(n, d):
\]

\#Precondition: \( n, d \) are natural numbers, \( d \neq 0 \)
\#Postcondition: \( r \) is a natural number such that

(a) \( r < d \), and
(b) \( n = kd + r \) for some \( k \) in \( N \)

\[
\begin{align*}
r & = n \\
\text{while } r \geq d \\
r & = r - d
\end{align*}
\]

return \( r \)

(a) Prove partial correctness:

(i) Determine a loop invariant and prove that it is correct.

(ii) Prove that \( \text{Precondition } \land \text{Termination } \rightarrow \text{Postcondition} \).

(b) Prove that the loop terminates.