1. Use a variation of simple induction to prove that for most natural numbers \( n \), any set of \( n \) elements has \( 2^{n-1} \) subsets with an odd number of elements.

2. We proved in class that for all natural numbers \( n \), \( 3^n \geq n^3 \). Your task is to complete the following alternative proof.

Define \( P(n) := "3^n \geq n^3" \).
As before, we prove \( \forall n. P(n) \) by a variation of simple induction.
Base case: you decide what base cases you need. We used 0, 1, 2, 3 in class, but this is a different proof, so perhaps you will need a smaller or larger number of base cases.
Let \( n \) be an arbitrary natural number that is at least as large as your largest base case. Assume \( P(n) \).
Goal: \( 3^{n+1} \geq (n+1)^3 \), or equivalently \( (n+1)^3 \leq 3^{n+1} \).
Expanding \( (n+1)^3 \) gives \( n^3 + 3n^2 + 3n + 1 \). Hence, the Goal is equivalent to:
NewGoal: \( n^3 + 3n^2 + 3n + 1 \leq 3^{n+1} \).
Prove NewGoal!