CSC 236H1 S February Midterm 2005
L0101
Duration — 50 minutes
Aids allowed: none

Student Number: |_|_|_|_|_|_|_|_|_|

Last Name: ____________________ **First Name:** ____________________

Do ***not*** *turn this page until you have received the signal to start.*
(Please fill out the identification section above,
and read the instructions below.) *Good Luck!*

This midterm consists of 3 questions on 5 pages (including this one). *When you receive the signal to start, please make sure that your copy is complete.*

For 1 bonus mark write your student number at the bottom of pages 2-5.

If you use any space for rough work indicate clearly what you want marked.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as "I do not know how to answer" followed or preceded by parts of a solution that have not been crossed off).

# 1:	______/ 6
# 2:	______/ 8
# 3:	______/10
TOTAL:	______/24

Question 1. [6 MARKS]

Suppose $x \in \mathbb{R}$ and

$$x + \frac{1}{x} \in \mathbb{N}.$$

Prove that for all $n \in \mathbb{N}$,

$$x^n + \frac{1}{x^n} \in \mathbb{N}.$$

Hint: what is $\left(x + \frac{1}{x}\right) \cdot \left(x^n + \frac{1}{x^n}\right)$?

Question 2. [8 marks]

Let f be defined for natural numbers n by

$$f(n) = \begin{cases} 0, & n = 0 \\ 2, & n = 1 \\ 3, & n = 2 \\ 23, & n = 3 \\ 236, & n = 4 \\ f(n-2) + 4f\left(\left\lfloor \frac{n}{2} \right\rfloor\right) - 3, & n \geq 5 \end{cases}$$

Prove that $f(n) \geq n^2$ for all $n \geq 3$.

(Note: you may not use the "Master Theorem")

Question 3. [10 MARKS]

Consider the following algorithm.

```
// Pre: n is a natural number, n ≥ 1
//      list is an array of n real numbers
a := 0
i := 0
while i ≠ n
  a := a + (list[i] - a)/(i + 1)
  i := i + 1
// Post: a is the average of the numbers in list:
//         (list[0] + ... + list[n-1]) / n
```

Part (a) [2 MARKS]

Express each of a_0, a_1, a_2 and a_3 non-recursively.

Part (b) [2 MARKS]

Complete the following loop invariant for the algorithm:

Let $I(k)$ be: $i_k = k$ and

Part (c) [6 MARKS]

Prove the partial correctness of the algorithm.

Total Marks = 24