

CSC236H1S October Midterm 2004

L5101

Duration — 50 minutes

Student Number: _____

Aids allowed: One 8.5x11" sheet of paper with anything on it.

Family Name: _____

Given Name: _____

Tutorial Section:
(circle one)

BA 2179
Yilan Gu

BA 2159
Zhao Dan

*Do **not** turn this page until you have received the signal to start.*
(Please fill out the identification section above,
and read the instructions below.) *Good Luck!*

This midterm consists of 4 questions on 5 pages (including this one).
When you receive the signal to start, please make sure that your copy is complete.

1: _____/10

For 1 bonus mark write your student number at the bottom of pages 2-5.

2: _____/10

If you do any rough work, indicate clearly what you want marked.

3: _____/10

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as "I do not know how to answer" followed or preceded by parts of a solution that have not been crossed off).

4: _____/10

TOTAL: _____/40

Question 1. [10 MARKS]

Define f by $f(1) = 2$ and

$$f(n) = f(\lfloor \sqrt{n} \rfloor)^2 + 3f(\lfloor \sqrt{n} \rfloor), \quad n \geq 2.$$

Prove by induction that $f(n)$ is divisible by 5 for all $n \geq 2$.

Question 2. [10 MARKS]

A full tree is a binary tree where each node has exactly 0 or 2 children.

Define the height of a non-empty tree as the maximum number of edges on any path that starts at the root of the tree and ends at a leaf (in particular, the height of a tree with a single node is zero).

Prove by induction that the number of leaves in a full tree is at least the height of the tree.

Question 3. [10 MARKS]

Prove that the following program is partially correct:

```
r := 2
i := n
while i > 0
  r := 3 * r - 2
  i := i - 1
```

Precondition: $n \geq 0$

Loop invariant: $r = 3^{n-i} + 1$

Postcondition: $r = 3^n + 1$

Question 4. [10 MARKS]

Consider the following program for computing the remainder when dividing integer m by integer n .

```
r := m
while r >= n
    r := r - n
```

Part (a) [2 MARKS]

State the most general precondition for which this program terminates.

Part (b) [2 MARKS]

Prove that this program terminates given your precondition for termination.

Part (c) [2 MARKS]

Complete the following postcondition for this program:

$r \geq 0$ and

Part (d) [2 MARKS]

State the precondition for this program.

Part (e) [2 MARKS]

State a loop invariant for this program.

Total Marks = 40