Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: |_|_|_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): _____

First (Given) Name(s): _____

Tutorial Section: BA 1170
Philipp Hertel

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This test consists of 3 questions on 5 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete and write your student number at the bottom of every page, where indicated.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lecture, tutorial, or on assignments. You must justify all other facts required for your solution.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/10

2: _____/15

3: _____/20

TOTAL: _____/45

Good Luck!

Question 1. [10 MARKS]

Let $G = (V, E)$ be an undirected weighted graph, with cost $c(e)$ on each edge $e \in E$.

Part (a) [4 MARKS]

Suppose that T is a minimum spanning tree of G. Replace the cost $c(e)$ of each edge by $2 \cdot c(e)$, therby creating a new instance of the problem with the same graph but different costs. Prove or disprove that T must still be a minimum spanning tree for the new instance.

Part (b) [3 MARKS]

Prove or disprove that the execution of Kruskal's and Prim's algorithm on G will always output the same minimum spanning tree.

Part (c) [3 MARKS]

Suppose that G has a unique minimum spanning tree T. Prove or disprove: all the edge costs in G must be distinct.

Question 2. [15 MARKS]

Consider the following "fast-growing subset sum" problem.

Input:

- A list of positive numbers $0 < a_1 < a_2 < \ldots < a_n$, such that each a_i is strictly bigger that the sum of all smaller numbers on the list: $a_i > a_1 + a_2 + \ldots + a_{i-1}$;
- a number $C > 0$.

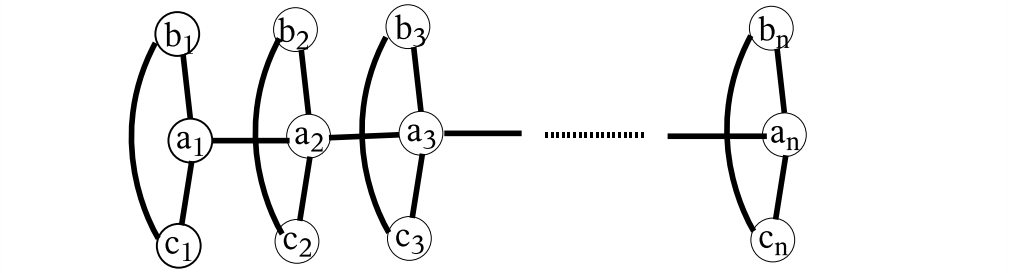
Output: A sublist of numbers $a_{i_1}, a_{i_2}, \ldots, a_{i_k}$ such that $a_{i_1} + a_{i_2} + \ldots + a_{i_k} = C$ if such a sublist exists, and 'no solution' otherwise.

For example, if the list is 2,3,7,14,26 then for $C = 23$ the algorithm should output $2 + 7 + 14$, and for input $C = 25$ it should output 'no solution'.

Write an efficient greedy algorithm to solve this problem, and prove that your algorithm is correct. (Your proof will be marked on its structure as well as its content.)

Question 3. [20 MARKS]

Let G be the graph with $3n$ vertices and $4n - 1$ edges presented on the picture below.



Weights $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n, c_1, c_2, \ldots, c_n$ are assigned to the vertices of G as on the picture. The goal of the question is to find an independent set of G that has the maximum total weight.

Part (a) [5 MARKS]

Show that the following greedy algorithm does not always return an independent set of G that has the maximum total weight.

$S := \emptyset$
while G is not empty:
    pick v_i in G with maximum weight
    remove v_i and its neighbours from G
    $S := S \cup \{v_i\}$
return S

Part (b) [15 MARKS]

Write a dynamic programming algorithm to solve this problem, and state the running time of your algorithm. (Your solution will be marked on its structure as well as its content.)

[More space on the next page.]

Question 3. (CONTINUED)

Part (b) (CONTINUED)

Total Marks = 45