Due: By 6:00pm on Wednesday 23 February, in the CSC 373H drop box in room BA 2220. **Worth**: 7.5%

4. [10 marks]
   Consider again the problem of multiplying two n-bit integers x, y, and suppose that n is a multiple of 3 (if not, we add one or two zeroes on the left to make this true). This time, suppose that we split each integer into three equal parts instead of two.

   (a) State the exact relationship between x and each of its three parts X_2, X_1, X_0 (*i.e.*, give an equation that involves x and X_2, X_1, X_0).

   (b) Write a divide-and-conquer algorithm to multiply two integers x, y based on this three-way split. Justify that your algorithm produces the correct answer, *i.e.*, show that the output of your algorithm is equal to $x \cdot y$.

   For full marks, your algorithm must run in time $o(n^2)$, *i.e.*, strictly less than $\mathcal{O}(n^2)$. Justify that this is the case by computing the running time of your algorithm (you may use the Master Theorem as long as you state clearly how it applies to your algorithm).

   (c) Is your algorithm faster or slower than the divide-and-conquer algorithm shown in class with a running time of $\Theta(n^{\log_2 3})$?