

Duration: **50 minutes**
 Aids Allowed: **NONE** (in particular, no calculator)

Student Number:

Last (Family) Name(s):

First (Given) Name(s):

*Do **not** turn this page until you have received the signal to start.*
 In the meantime, please read the instructions below carefully.

This term test consists of 3 questions on 8 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write “I don’t know” and nothing else — you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as “I don’t know” followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/11

2: _____/12

3: _____/10

TOTAL: _____/33

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 1. [11 MARKS]**Part (a)** [5 MARKS]

True or False: “For all languages $A, B \subseteq \Sigma^*$, if $A \leq_p B$, then $B \leq_p A$ ”? Prove your claim.

Part (b) [6 MARKS]

Prove that $NP = coNP$ iff there exists a language A that is both NP -complete and $coNP$ -complete.

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 2. [12 MARKS]

Give a detailed proof that the following language EQUIV is *coNP*-complete, including a verifier for showing that EQUIV belongs to *coNP*.

$\text{EQUIV} = \{ \langle F_1, F_2 \rangle : F_1 \text{ and } F_2 \text{ are equivalent propositional formulas, i.e., every assignment of values to the variables of } F_1 \text{ and } F_2 \text{ makes both formulas True or both formulas False} \}$

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 3. [10 MARKS]

Show that the following problem is polytime self-reducible. Include a clear argument that your algorithm is correct and runs within an appropriate time bound.

CNF-SAT search problem:

Input: A propositional formula F in CNF.

Output: A satisfying assignment for F , if one exists—the special value **None** otherwise.

CNF-SAT decision problem:

Input: A propositional formula F in CNF.

Question: Does F have some satisfying assignment?

Recall that a *satisfying assignment* for propositional formula F is an assignment of values to the variables of F that makes F true. Also, note that the input for the decision problem must be a syntactically correct formula in CNF—written using only variables, negated variables, and the connectives “ \vee ” and “ \wedge ”.

On this page, please write nothing except your name.

Last (Family) Name(s): _____

First (Given) Name(s): _____