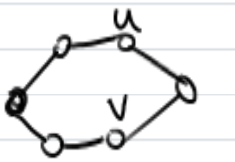
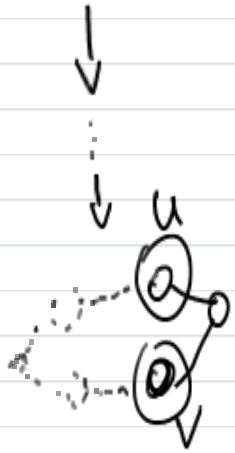
Counter examples

Example 1 (for question 3)

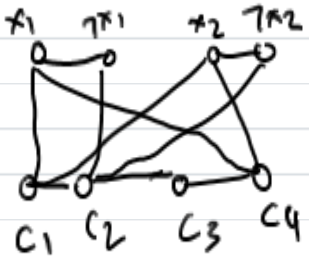
$k=2$





If the algorithm removes vertices 1-by-1 then it may end up finding these two vertices

Example 2 (for question 1)



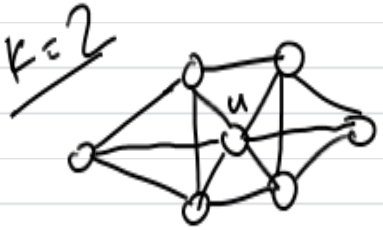
The example is for 2SAT (similarly for 3SAT)

use C_5 for "gap" (similarly without)

Example 3 (for question 1)

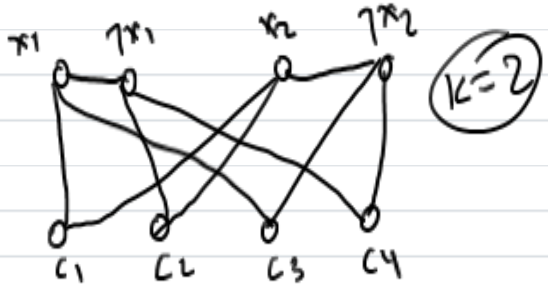
Similar to example 2 (without the linear array gadget - remove edges)

Example 4 (for question 3)



Consider the alg.
starting with u
(and removing this
and all its neighbors)

Example 5 (for question 1)



(again the example for 2SAT
easy to generalize for 3SAT)
the formula is unsat, however there is
CS of size 2 (e.g x_1, x_2)