

Duration: **50 minutes**
 Aids Allowed: **NONE** (in particular, no calculator)

Student Number:

Last (Family) Name(s):

First (Given) Name(s):

*Do **not** turn this page until you have received the signal to start.*
 In the meantime, please read the instructions below carefully.

This term test consists of 3 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write “I don’t know” and nothing else — you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as “I don’t know” followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/11

2: _____/11

3: _____/11

BONUS

MARKS: _____/ 6

TOTAL: _____/33

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 1. [11 MARKS]**Part (a)** [6 MARKS]

Prove that the following language is *NP*-complete.

$$\text{BA}_{\text{NTM}} = \{ \langle M, x, t \rangle : M \text{ is a NTM that accepts input } x \text{ within } t \text{ steps} \}$$

Part (b) [5 MARKS]

Prove or disprove: “For all languages A, B , if $A \leq_p B$ and $B \in \text{coNP}$, then $A \in \text{coNP}$.”

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 2. [11 MARKS]

Prove that the following language is *NP*-complete.

SMALLSUBSETSUM (SSS) = $\{ \langle S, t \rangle : S = \{x_1, \dots, x_n\}$ is a non-empty set of positive integers and t is a positive integer such that some subset of S of size **at most** $n/2$ adds up to t (i.e., $\exists S' \subseteq S, |S'| \leq n/2 \wedge \sum_{x \in S'} x = t$) }

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 3. [11 MARKS]

Prove that the following SUBSETSUMSEARCH problem is polytime self-reducible.

SUBSETSUMSEARCH problem:

Input: A non-empty set of positive integers S and an integer $t > 0$.

Output: A subset of S whose sum equals t (or “None” if there is no such subset).

SUBSETSUM decision problem:

Input: A non-empty set of positive integers S and an integer $t > 0$.

Question: Does S contain some subset whose sum equals t , *i.e.*, $\exists S' \subseteq S, \sum_{x \in S'} x = t$?

Use this page for rough work — clearly indicate any section(s) to be marked.

Bonus. [6 MARKS]

WARNING! This question is difficult and will be marked harshly: credit will be given only for *significant* progress toward a correct answer (in particular, “I don’t know” will be worth zero). Please attempt this only *after* you have completed the rest of the test.

Give a specific example of a language that is both *NP*-hard and *coNP*-hard. Justify that your language has both properties. What would you be able to conclude if your language were *NP*-complete or *coNP*-complete? Give the strongest consequence possible and explain.

On this page, please write nothing except your name.

Last (Family) Name(s): _____

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