Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: |_|_|_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): _____

First (Given) Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below *carefully*.

This term test consists of 4 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, complete the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, assignments, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do—part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for the question (or part) if you write "I don't know" and nothing else—you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/ 7

2: _____/ 9

3: _____/12

4: _____/12

TOTAL: _____/40

Good Luck!

OVER...

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 1. [7 MARKS]

Answer each part for an arbitrary function $f : \mathbb{N} \mapsto \mathbb{R}^+$ such that $f(n) \geqslant n$.

Part (a) [3 MARKS]

Give the definition of $NTIME(f(n))$.

Part (b) [2 MARKS]

Give a specific function $g(n)$ (in terms of $f(n)$) such that $NSPACE(f(n)) \subseteq SPACE(g(n))$.

Part (c) [2 MARKS]

Give a function $h(n)$ (in terms of $f(n)$) such that $SPACE(f(n)) \subseteq TIME(h(n))$.

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 2. [9 MARKS]

Part (a) [3 MARKS]

Give the definition of P, and one example of a language in P that is most likely *not* in L.

Part (b) [2 MARKS]

Give one example of a language in NP that is most likely *not* in P.

Part (c) [4 MARKS]

Give the definition of *PSPACE*, and one example of a language in *PSPACE* that is most likely *not* in *NP*.

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 3. [12 MARKS]

Show that language FEW-SAT defined below is *coNP*-complete.

FEW-SAT = $\{ \langle F \rangle : F$ is a propositional formula that can be satisfied by at most one setting of its variables, *i.e.*, either F is false no matter how its variables are set, or there is exactly one way to set F's variables in order to make F true $\}$

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 4. [12 MARKS]

In the "Price Is Right" optimization problem, you are given a set of positive integer prices $P = \{p_1, \ldots, p_m\}$ and a positive integer target T, and you must find a subset of the prices $P' \subseteq P$ such that the total of P' is as large as possible without going over T, *i.e.*, $\sum_{p \in P'} p$ is maximal and $\sum_{p \in P'} p \leqslant T$. The corresponding language PIR is defined as follows.

$$\text{PIR} = \left\{ \langle P, T, M \rangle : P \text{ is a set of positive integers and } T, M \text{ are positive integers such that there is a subset } P' \subseteq P \text{ whose sum satisfies } M \leqslant \sum_{p \in P'} p \leqslant T \right\}$$

For example, for $P = \{5000, 10000, 21000, 38000, 67000\}$ and $T = 52000$, the subset $P' = \{10000, 38000\}$ has sum 48000 and all other subsets of P have a sum smaller than 48000 or greater than 52000. This means that, for example, $\langle P, T, M = 45000 \rangle \in \text{PIR}$ but $\langle P, T, M = 50000 \rangle \notin \text{PIR}$.

Show that the "Price Is Right" optimization problem is self-reducible. (Use n to represent the size of input $\langle P, T, M \rangle$, *i.e.*, the total number of bits required to represent all the numbers $p_1, \ldots, p_m, T, M$.)

On this page, please write nothing except your name.

Last (Family) Name(s): _____

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