

Worth: 2%

Due: By 6pm on Wednesday 21 October

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

Consider a function $f(n)$ that satisfies the following recurrence relation:

$$f(n) = \begin{cases} 1 & \text{if } n = 1 \text{ and } n = 0, \\ 2f(\lfloor n/3 \rfloor) + \lceil \sqrt{n} \rceil & \text{if } n > 1. \end{cases}$$

1. Apply the Master Theorem to find a tight bound on the value of $f(n)$ —show your work, *i.e.*, justify that the theorem applies and explain what values the various parameters take.
2. Apply Repeated Substitution to try to find an “exact” expression for the value of $f(n)$ —show your work, *i.e.*, explain what you are doing at each step of your solution, and why.

Recall the following facts about logarithms (in your solution, you may make use of any of these facts without proof): for all real numbers $b > 1$, $x > 0$, $y > 0$,

- $x^{\log_b y} = y^{\log_b x}$
- $\log_b(x^y) = y \log_b x$
- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b(x/y) = \log_b x - \log_b y$