

**Worth:** 2%**Due:** By 6pm on Wednesday 14 October

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

Consider the following algorithm—it is not a realistic algorithm for the task that it carries out, but we are concerned only with its running time in this Exercise.

```
SUM( $A, b, e$ ):  
  if  $b == e$ :  
    return  $A[b]$   
  # The rest executes only when  $b != e$ .  
   $m = (b + e)/2$   
   $s = 0$   
  for  $i = m + 1, \dots, e$ :  
     $s += A[i]$   
  return  $s + \text{SUM}(A, b, m)$ 
```

1. Let  $T(n)$  denote the worst-case running time of SUM on inputs of size  $n$ . Write a recurrence relation satisfied by  $T$ . Make sure to define  $n$  precisely (as a function of the algorithm's parameters) and justify that your recurrence is correct (by referring to the algorithm). In particular, describe clearly what “steps” you are counting.
2. Write a detailed proof that  $T(n) \in \Omega(n)$ , based on your recurrence relation from the previous part. Note that you should **not** attempt to “solve” the recurrence in order to answer this question—“solving” the recurrence is done in order to know what function to use as a bound on  $T(n)$ , and we've already provided you with that function in this question.