Worth: 2% **Due:** By 6pm on Wednesday 23 September

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

In particular, keep in mind that the main point of this exercise is to get you to practice writing proofs by induction. This means that you will be marked mostly on the structure of your proofs and how they were written up—in other words, having the right idea but writing it up poorly will be worth less than having the wrong idea but writing it up well...

1. For each statement below, give *two* different inductive proof structures that could be used to prove the statement. Fill in as much of each proof structure as possible, without making any assumption about predicate P.

   (a) For all *even* integers $x \in \mathbb{Z}$, $P(x)$.

   (b) For all n that are powers of 10, $P(n)$. (Recall that n is a power of 10 iff $\exists k, n = 10^k$.)

   (c) For all S that are finite sets of integers, $P(S)$.

2. Let **B** be the set of strings consisting of properly nested brackets, defined recursively as follows:

   - $\varepsilon \in \mathbf{B}$,
   - if $x, y \in \mathbf{B}$, then $[x] \in \mathbf{B}$ and $xy \in \mathbf{B}$,
   - nothing else belongs to **B**.

   Prove that $\forall s \in \mathbf{B}$, the number of left brackets "[" in s equals the number of right brackets "]" in s.

   Food for thought: How might you go about proving that an illegal string of brackets (such as "]]" or "] [") does *not* belong to **B**?