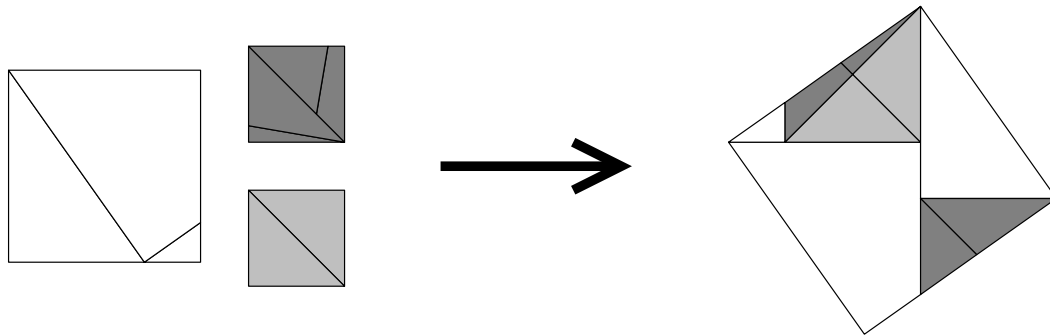


Worth: 2%**Due:** By 6pm on Wednesday 16 September

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

In particular, keep in mind that the main point of this exercise is to get you to practice writing proofs by induction. This means that you will be marked mostly on the structure of your proofs and how they were written up—in other words, having the right idea but writing it up poorly will be worth less than having the wrong idea but writing it up well...

1. Write a detailed proof that for all natural numbers n , $10^n - 1$ is an integer multiple of 9.
2. Let $P(n)$ be the statement: “**any** n squares of various sizes can be dissected (cut in a **finite number of straight lines**) and rearranged to form one large square.” For example, in the picture below, each of the three squares on the left can be cut as indicated and the pieces can be moved and rotated to form the square on the right.



Obviously $P(1)$ holds. You may use without proof the fact that $P(2)$ holds.

Write a detailed proof that $P(n)$ holds for all natural numbers $n \geq 1$.

Bonus: Prove the case $P(2)$.