

CONTENTS

- 1 INTRODUCTION 5
 - 1.1 What’s csc165 about? 5
 - 1.2 Human versus technical communication 7
 - 1.3 Problem-solving 8
 - 1.4 Inspirational puzzles 9

- 2 QUANTIFICATION, IMPLICATION, AND SYMBOLS 13
 - 2.1 Universal quantification 13
 - 2.2 Existential quantification 14
 - 2.3 Properties, sets, and quantification 15
 - 2.4 Sentences, statements, and predicates 16
 - 2.5 Implications 17
 - 2.6 More symbols 18
 - 2.7 Implication in everyday English 20

- 3 LOGICAL CONNECTIVES 23
 - 3.1 Universal quantification and implication again 23
 - 3.2 Vacuous truth 24
 - 3.3 Equivalence 24
 - 3.4 Restricting domains 25
 - 3.5 Conjunction (And) 25
 - 3.6 Disjunction (Or) 26
 - 3.7 Negation 26
 - 3.8 Symbolic grammar 27
 - 3.9 Truth tables 28
 - 3.10 Tautology, satisfiability, unsatisfiability 28
 - 3.11 Logical “arithmetic” 29
 - 3.12 DeMorgan’s Laws 29
 - 3.13 Implication, bi-implication, with \neg , \vee , and \wedge 30
 - 3.14 Transitivity of universally-quantified implication 30
 - 3.15 Summary of manipulation rules 30
 - 3.16 Multiple quantifiers 31
 - 3.17 Mixed quantifiers 31

- 4 PROOFS 35
 - 4.1 What is a proof? 35
 - 4.2 Direct proof of universally-quantified implication 36
 - 4.3 Hunting the elusive direct proof 36

4.4	An odd example of direct proof	37
4.5	Another example of direct proof	38
4.6	Direct proof of universally-quantified predicate	39
4.7	Indirect proof of universally-quantified implication	39
4.8	Proof by contradiction	39
4.9	Direct proof structure of the existential	40
4.10	Multiple quantifiers, implications, and conjunctions	40
4.11	Example of proving a statement about a sequence	41
4.12	Example of disproving a statement about a sequence	42
4.13	Non-boolean function example	43
4.14	Substituting known results	44
4.15	Proof by cases	44
4.16	Proving \vee using cases	46
4.17	Building formulae and taking formulae apart	47
4.18	Summary of inference rules	48
5	ASYMPTOTIC NOTATION	53
5.1	Run time and constant factors	53
5.2	Asymptotic notation: Making Big-O precise	54
6	CORRECTNESS, RUNNING TIME OF PROGRAMS	63
6.1	Binary (base 2) notation	63
7	NUMERICAL SYSTEMS	71
7.1	Floating-point systems	71
7.2	Expressing real numbers	72
7.3	Computing with floating point numbers	73
7.4	Stability	75
7.5	Conditioning	76
7.6	Truncation	77
7.7	Error summary	77