

Generalities

Work with 1–3 other students, and choose one of your group as the recorder (who will keep a written record of your progress). Make a start on the problem for about 5–10 minutes, then stop and review any choices you’ve made about how to proceed. I have suggested a couple of approaches to consider on the back of this sheet, but you should only look at these after you have made a good start on the problem. Organize your discussion following the “UPEEG” method:

Understand the problem—in particular, are there various ways to represent it?

Plan one approach—or two! What is the outcome you expect from your plan?

Execute your plan.

Evaluate your progress—figure out when and how you became stuck (if you did), and what insights (if any) represented a breakthrough.

Generalize your results—think of various ways in which the problem could be generalized, and whether or not your approach(es) would still work.

The problem

Suppose you have tiles that are 1×2 rectangles, and you have a walk that is 2×3 units long. There are three ways to tile your walk: three vertical tiles, one vertical tile followed by two horizontal tiles, or two horizontal tiles followed by one vertical tile.

How many different ways are there to tile a 2×4 walk, or (more generally) a $2 \times n$ walk (where n is any natural number)? Can you come up with a general procedure that works?

What happens if your tiles are 1×3 , and you have a walk that is $3 \times n$ units long? What about other tile and walk dimensions?

Hint 1—organize: Organize your counting according to the orientation of the left-most tile.

Hint 2—think recursively: Is there a simple, exploitable relationship between the answer for n and the answer for a smaller value of the parameter?