

## Generalities

Work with 1–3 other students, and choose one of your group as the recorder (who will keep a written record of your progress). Make a start on the problem for about 5–10 minutes, then stop and review any choices you’ve made about how to proceed. I have suggested a couple of approaches to consider on the back of this sheet, but you should only look at these after you have made a good start on the problem. Organize your discussion following the “UPEEG” method:

**Understand** the problem—in particular, are there various ways to represent it?

**Plan** one approach—or two! What is the outcome you expect from your plan?

**Execute** your plan.

**Evaluate** your progress—figure out when and how you became stuck (if you did), and what insights (if any) represented a breakthrough.

**Generalize** your results—think of various ways in which the problem could be generalized, and whether or not your approach(es) would still work.

## The problem

The list of positive integers that add up to 1 is [1], and the product of this list (if you allow unary products) is also 1.

There are two lists of positive integers that add up to 2, and they yield two different products: [2] (with product 2) and [1, 1] (with product 1).

There are several lists of positive integers that add up to 3, and they yield several different products: [3] (with product 3), [2, 1] and [1, 2] (with product 2), and [1, 1, 1] (with product 1).

In general, if  $n$  is a positive integer, what is the maximum product that can be formed of a list of positive integers that sum to  $n$ ?

**Hint 1—organize:** Even for fairly small numbers  $n$ , you will have quite a large list of numbers that sum to  $n$ . Do you have some organizing principle for your lists?

**Hint 2—economize:** Are there some lists that you can remove from consideration right away?