

## Generalities

Work with 1–3 other students, and choose one of your group as the recorder (who will keep a written record of your progress). Make a start on the problem for about 5–7 minutes, then stop and review any choices you’ve made about how to proceed. I have suggested a couple of approaches to consider on the back of this sheet, but you should only look at these after you have made a good start on the problem. For this problem, use only one hint at a time, since they lead to different solution paths. Use the following headings to organize the discussion:

- Understand the problem—in particular, are there various ways to represent it?
- Devise a plan—or two! What is the “best case” result you expect from your plan?
- Carry out the plan—and verify it.
- Look back, figure out when and how you became stuck, and what insights represented a breakthrough.

## The problem

You are sitting in front of two drawers. The left drawer contains 64 pennies, the right drawer contains nothing. Can you arrange things so that one of the drawers has exactly 48 pennies, using only the following two operations?

**L:** If the left drawer has an even number of pennies, transfer half of them to the right drawer (if the left drawer has an odd number of pennies, operation **L** is disallowed).

**R:** If the right drawer has an even number of pennies, transfer half of them to the left drawer (if the right drawer has an odd number of pennies, operation **R** is disallowed).

What about arranging things so that one of the drawers contains other numbers in the range  $[0, 64]$ ? What about starting with a different number of pennies in the left drawer?

**Hint 1—work backwards:** Imagine you have already carried out steps that give you the desired number of pennies in one drawer. What would the **second-last** step be (the step just before the successful step)?

**Hint 2—smaller cases:** Is there any connection between the steps to get 24 in a drawer when you start with 64, and getting 12 in a drawer when you start with 32?

**Hint 3—draw a picture:** Draw a tree diagram of all the possible results (amounts of pennies in each drawer). Try to be systematic.