1. Prove or disprove that the set $S_0 = \{ f : \mathbb{Z} \rightarrow \mathbb{Z} \}$ is countable.
2. Prove or disprove that the set $S_1 = \{ S \subseteq \mathbb{N} : \text{size}(S) \text{ is finite} \}$ is countable.
3. Prove or disprove that the set $S_2 = \{S \subseteq \mathbb{N} : \text{size}(S) \text{ is infinite} \}$ is countable.
4. Prove or disprove that the set $S_3 = \{(a, b) : a \in \mathbb{N}, b \in \mathbb{N}\}$ is countable.