Duration: **50 minutes**
Aids Allowed: **none**

Student Number: _____

Family Name(s): _____

Given Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below *carefully*.

This term test consists of 3 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do—part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write "I don't know" and nothing else—you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

# 1:	_____/12
# 2:	_____/15
# 3:	_____/ 9
BONUS MARKS:	_____/ 5
TOTAL:	_____/36

Good Luck!

OVER...

Use this page for rough work—clearly indicate any section(s) to be marked.

Question 1. [12 MARKS]

Part (a) [2 MARKS]

Complete the following **formal definition** by checking all that apply:

Language A is "mapping reducible" to language B, written $A \leqslant_m B$, if...

- ☐ ...it is possible to use a recognizer for B in order to recognize A.
- ☐ ...there is a computable function $f : \Sigma^* \to \Sigma^*$ such that $\forall w \in \Sigma^*$, $w \in A \Leftrightarrow f(w) \in B$.
- ☐ ...it is possible to use a recognizer for A in order to recognize B.
- ☐ ...there is a computable function $f : \Sigma^* \to \Sigma^*$ such that $\forall w \in \Sigma^*$, $w \in B \Leftrightarrow f(w) \in A$.

Part (b) [5 MARKS]

Prove or disprove: "for all recognizable languages L, $A_{\text{TM}} \leqslant_m L$".

Part (c) [5 MARKS]

Prove or disprove: "for all recognizable languages L, $L \leqslant_m A_{\text{TM}}$".

Use this page for rough work—clearly indicate any section(s) to be marked.

Question 2. [15 MARKS]

State whether the following language is decidable, undecidable but recognizable, undecidable but co-recognizable, or unrecognizable and co-unrecognizable, then prove your claim.

$$L_0 = \{\langle M\rangle : M \text{ is a TM that accepts at least two different inputs and that rejects at least two different inputs (also different from the inputs accepted by } M\text{, obviously)}\}$$

Use this page for rough work—clearly indicate any section(s) to be marked.

Question 3. [9 MARKS]

State whether each language below belongs to P, NP, or $coNP$. Make the strongest claims possible and justify your answers (there is no need to argue that languages in NP or $coNP$ do not belong to P).

For this question, an "independent set" in an undirected graph G with vertex set V and edge set E is some subset of vertices $I \subseteq V$ such that G contains **no** edge between any two vertices in I.

Part (a) [3 MARKS]

$\textsc{Indep}_1 = \{\langle G, k\rangle : G$ is an undirected graph and k is a positive integer such that G contains **some** independent set of size k **or less**$\}$

Part (b) [3 MARKS]

$\textsc{Indep}_2 = \{\langle G, k\rangle : G$ is an undirected graph and k is a positive integer such that G contains **some** independent set of size k **or more**$\}$

Part (c) [3 MARKS]

$\textsc{Indep}_3 = \{\langle G, k\rangle : G$ is an undirected graph and k is a positive integer such that G contains **no** independent set of size k **or more**$\}$

Use this page for rough work—clearly indicate any section(s) to be marked.

Bonus. [5 MARKS]

WARNING! This question is difficult and will be marked harshly: credit will be given only for making *significant* progress toward a correct answer (in particular, "I don't know" will be worth zero). Please attempt this only *after* you have completed the rest of the test.

Prove that a language A is recognizable iff $A = \{x : \exists y, \langle x, y\rangle \in B\}$ for some decidable language B.

On this page, please write nothing except your name.

Family Name(s): _____

Given Name(s): _____