Duration: **50 minutes**
Aids Allowed: **none**

Student Number: |_|_|_|_|_|_|_|_|_|_|_|

Family Name(s): _____

Given Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below *carefully*.

This term test consists of 3 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do—part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write "I don't know" and nothing else—you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/12

2: _____/12

3: _____/12

BONUS MARKS: _____/ 5

TOTAL: _____/36

Good Luck!

OVER...

Use this page for rough work—clearly indicate any section(s) to be marked.

Question 1. [12 MARKS]

Write a Turing Machine that increments a natural number in dyadic notation. More precisely, when started in configuration "$\sqcup\ q_0\ x$", where $x \in \{1,2\}^*$, your TM will eventually reach configuration "$\sqcup\ q_A\ x'$", where $\mathrm{val}(x') = \mathrm{val}(x) + 1$. For example, initial configuration "$\sqcup\ q_0\ \sqcup$" goes to accepting configuration "$\sqcup\ q_A\ 1$", initial configuration "$\sqcup\ q_0\ 1$" goes to accepting configuration "$\sqcup\ q_A\ 2$", initial configuration "$\sqcup\ q_0\ 2$" goes to accepting configuration "$\sqcup\ q_A\ 11$", *etc.*

Give a high-level description followed by a formal-level description, by filling in the transition table below (put a dash "—" in any box where a transition is superfluous). Provide descriptive comments to explain the purpose of each state. You may not require every state below—simply leave transitions blank for unused states—and for full marks, do *not* add any more states. (HINT: Handle the general case first and special cases last.)

	$\sqcup$	1	2	comment/purpose
q_0				
q_1				
q_2				
q_3				
q_4				
q_5				

Use this page for rough work—clearly indicate any section(s) to be marked.

Question 2. [12 MARKS]

An "Increasing Movement Turing Machine" (IMTM) is similar to a regular TM except that the tape is infinite in both directions and during transition number k, the head moves k squares to the left or right, *i.e.*, even though each movement is specified by a single "L" or "R", the head moves 1 square to the left or 1 square to the right during the first transition, 2 squares to the left or 2 squares to the right during the second transition, *etc.*

Write a detailed, implementation-level proof that every recognizable language can be recognized by an IMTM. You may make use of the fact that TMs with a 2-way infinite tape are equivalent to regular TMs. (HINT: Don't panic! We're asking you to prove only the *easy* direction...)

Use this page for rough work—clearly indicate any section(s) to be marked.

Question 3. [12 MARKS]

For any language $L \subseteq \Sigma^*$, consider the language

$$L^+ = \{w \in \Sigma^* : L \text{ contains } \textbf{some} \text{ prefix of } w\}.$$

Give a detailed, implementation-level proof that if L is recognizable, then L' is recognizable.

Use this page for rough work—clearly indicate any section(s) to be marked.

Bonus. [5 MARKS]

WARNING! This question is difficult and will be marked harshly: credit will be given only for making *significant* progress toward a correct answer (in particular, "I don't know" will be worth zero). Please attempt this only *after* you have completed the rest of the test.

Write a detailed, implementation-level proof that every language recognized by an IMTM is recognizable (IMTMs are defined in Question 2).

On this page, please write nothing except your name.

Family Name(s): _____

Given Name(s): _____